PRESSURE DROP IN TWO-PHASE FLOW

By

SAEDELDEEN AHMAD AKASHAH

Bachelor of Science in Chemical Engineering
Tri-State University
Angola, Indiana
1972

Master of Science
Oklahoma State University
Stillwater, Oklahoma
1978

Submitted to the Faculty of the Graduate College
of the Oklahoma State University
in partial fulfillment of the requirements
for the Degree of
DOCTOR OF PHILOSOPHY
December, 1980

PRESSURE DROP IN TWO-PHASE FLOW

Thesis Approved:

Thesis Adviser

Dean of the Graduate College

PREFACE

A study of some of the methods for predicting pressure drop resulting from two-phase flow in pipelines was conducted. A computer package was developed containing the most widely used methods for horizontal and vertical two-phase flow coupled with accurate physical properties prediction methods. The package is self-contained and can be used for design and/or operation of a pipeline.

I would like to express my deepest appreciation for the help, guidance and concern that my Adviser, Dr. R. N. Maddox, has shown me throughout my work at Oklahoma State University. I am truly grateful to Dr. J. H. Erbar for his help and assistance in developing this work. I would like to thank the other members of my Advisory Committee, Dr. K. J. Bell and Dr. G. J. Mains, for their valued suggestions.

I thank the University of Kuwait for providing me with financial support throughout my graduate studies.

My family, and especially my wife, Mary, deserve my appreciation for their constant support and encouragement.

Finally, I would like to dedicate this work to Dr. M. Y. Shana'a for encouraging me to pursue graduate work - Misery Loves Company.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure | Page

NOMENCLATURE

English Letters

D	- diameter - cm or ft
E_ℓ	- liquid holdup fraction for the Duns and Ros method
f	- friction factor
F_1, F_2, F_3, F_4, F_5, F_6, F_7	- nondimensional function for the Duns and Ros method
g_o	- acceleration of gravity, 9.8 N, 32.2 ft/sec^2
g_c	- universal conversion factor
G	- nondimensional total pressure gradient
G_{st}	- non dimensional static pressure gradient
G_{fr}	- nondimensional frictional pressure gradient
$H_L(0)$	- horizontal liquid holdup

English Letters (Continued)

$H_L(\theta)$ - inclined liquid holdup

N - liquid velocity number

$\cancel{N_b}$ - bubble number

N_d - diameter number

N_{gv} - gas velocity number

N_ℓ - liquid viscosity number

N_m - mist flow number

N_s - slug flow number

P - pressure

R_e - Reynolds Number

R - density ratio

R_N - gas velocity number (Duns and Ros)

R_{Lq} - liquid holdup

S - correlating parameter for the slip velocity

U - overall heat transfer coefficient, Kw/m^2K, $Btu/hrft^2F^o$

V - velocity, cm/sec or ft/sec

W_o - Weber Number

Greek Letters

ε - roughness factor, cm or ft

λ - volumetric liquid fraction

σ - surface tension, $dynes/cm^2$

μ - absolute viscosity, centipoise, lb/ft sec

ρ - density, gm/cc or lb/ft^3

Subscripts

o	- single phase
ℓ	- liquid
ns	- non-slip
s	- slip
tp	- two-phase
v	- vapor
w	- wall

CHAPTER I

INTRODUCTION

Two-phase flow is the simultaneous flow of two phases in a single pipeline. In this work, the two phases referred to are gas and liquid. The applications of two-phase flow in industry are many and the literature contains numerous articles and books on the subject.

One common feature of existing methods is the lack of reliable methods for predicting thermodynamic and physical properties. The accuracy and consistency of the results are, therefore, doubtful. The need for combining a reliable equation of state with some of the methods of two-phase flow was the reason for undertaking this work.

The combination of an equation of state with two-phase predictive methods yields a powerful tool for designing and/or operating a pipeline.

An equation of state (SRK) was combined with physical properties predictive methods and several methods of calculating two-phase flow in a modular computer program capable of predicting pressure drop in a given pipeline. It can calculate pressure drops for horizontal, inclined, and vertical flow. It is also capable of predicting pressure drop in adiabatic or non-adiabatic flow.

CHAPTER II

LITERATURE REVIEW

The study of two-phase flow has been reported widely in the literature. The sheer volume of what has been written, the diversity of the approaches used, and the wide range of specific applications make review of the literature in this area difficult. Many studies have had as their objective a review of the methods used in two-phase flow. The work of DeGance and Atherton (1-8) is particularly good for an overall view of gas-liquid two-phase flow.

The publications reviewed in this chapter are limited to those with proven practical application, i.e., used in the industry, or of historical value in the development of two-phase flow studies, or both.

The first method for the prediction of pressure drop for two-phase flow in pipes was that of Lockhart and Martinelli (9). The correlation obtained was based upon experimental data for the flow of air and various liquids in pipes ranging in diameter from 0.0586 inches to 1.017 inches. The approach was purely empirical and resulted in a correlating parameter that is the square root of the ratio of the pressure drops that would result if each phase occupied the entire conduit. The correlating parameter was then used to obtain a function that would predict two-phase pressure drop from the single-phase pressure drop. This, in turn, is a function of the correlating parameter and the type of flow that exists during the simultaneous flow of both phases. Lockhart and Martinelli proposed the following flow mechanisms:

1. Turbulent liquid and turbulent gas flow
2. Viscous liquid and turbulent gas flow
3. Turbulent liquid and viscous gas flow
4. Viscous liquid and viscous gas flow.

They presented the parameter and function in graphical forms which were unusable for computer application. DeGance and Atherton (4) curve-fitted the graphs and obtained equations that can be used.

The data for developing the correlation were limited (diameter 0.0586 to 1.017 inches) and confined to two components (air-water, air-benzene, air-kerosene). Only isothermal flow was considered. In spite of all of this, the correlation proved to be of great practical use and is still used today. More importantly, subsequent methods and correlations followed this general approach to predict pressure drop in two-phase flow.

A recent article (10) proposed a nomograph based on the Lockhart-Martinelli equation. Besides being cumbersome, the nomograph proved to be inaccurate for a test case. The difference between the nomograph and Lockhart and Martinelli's method was 40 percent. This is due in part to the many lines that have to be drawn in order to obtain the pressure drop. Any small deviation in the slope or intercept of a line can change the result by an order of magnitude.

Baker (11,12) expanded on the Lockhart and Martinelli work by introducing the effect of flow patterns, inclined flow, temperature change, and pipeline efficiency. Baker retained the correlating parameters of Lockhart and Martinelli but introduced new parameters to determine the flow regime. The method of calculating the single-phase pressure drop and the correlating function are dependent on the flow regime.

Although there have been numerous publications dealing with pressure drop in gas-liquid flow since the Baker correlation was introduced, a major contribution was the result of a project funded by the American Gas Association and the American Petroleum Institute (13,14). Data for a wide range of conditions were collected from the literature. Attempts were made to evaluate the data as to accuracy, range and reliability. Existing correlations were then tested against the evaluated data. The correlations tested provided a starting point for developing an improved method for predicting two-phase pressure drop.

By applying similarity analysis they developed a liquid holdup correlation for horizontal flow that combines all types of flow in a single graph. With the application of a three dimensional table reading subroutine the correlation could be used quite readily in a computer program.

The AGA-API project recommends the Flanigan (14) correlation for two-phase flow in inclined pipelines.

The results of the project were presented in a design manual (14) and represent a practical approach to computer calculation. The generality of the method offers reasonable accuracy with simple application.

Duns and Ros (15) developed a calculation procedure for the prediction of pressure variation in oil wells and gas/condensate flow over a wide range of field operating conditions. The correlations are complicated and there is a need for computerized calculations. However, the correlations were presented in graphical form and are hardly useful in computer applications.

Duns and Ros proposed three regions for vertical two-phase flow:

1. Region I which includes bubble flow, plug flow and part of froth flow.

2. Region II which includes froth flow and slug flow.

3. Region III which includes mist flow, with a transition region existing between it and Region II.

The dependency of liquid holdup and friction upon the velocities of gas and liquid, pipe diameter, liquid viscosity, liquid density, and surface tension led Duns and Ros to the development of four dimensionless numbers that are used to determine the three regions.

The same approach was used by Orkiszewski (16,17) although four regions were proposed.

1. Region I consisting of bubble flow

2. Region II containing slug flow

3. Region III containing annular-slug transition flow

4. Region IV consisting of annular mist flow.

The method was produced by applying six methods to field data from twenty-two wells, and then modifying existing methods.

The method is valid for pipes ranging in diameter from three to eight inches. The use of dimensionless numbers to determine the flow regions is very similar to the procedure Duns and Ros. DeGance and Athertone (5) consider this method to be the most accurate method for pure vertical flow in small diameter pipes.

For inclined pipes, the method of Beggs and Brill (18,19) can be used for the specific evaluation of pipelines passing through hilly terrain. The method is based upon experimental measurements using air and water. An updated version of the test system was reported in 1979 (2), utilizing basically the same concept although the updated system used natural gas and water.

The basic elements of Beggs and Brill's method are a correlation of the angular liquid holdup as a function of horizontal holdup and a correlation of the two-phase friction factor as a function of single-phase friction factor. DeGance and Atherton (5) curve-fitted the graphical form of the correlations making the use of the method in a computer program possible.

Recently the Beggs and Brill method came under attack from Danesh (21). Using a gas-condensate pipeline data Danesh reported negative values and values greater than one for the liquid holdup predicted by Beggs and Brill. Danesh concluded that since the correlations are based upon an air-liquid mixture, the method over-predicts the horizontal holdup for high pressure gas-condensate pipeline. The effects of physical properties are not considered in the prediction of horizontal liquid holdup, although they are included in the parameters that are used to determine the region of flow. An unsuccessful attempt was made (22) to obtain data from Dr. Danesh to test on other methods.

Although there are several other sources worthy of consideration (23,24,25,26), the most important work in this writer's opinion is that of Erbar and Maddox (27). The idea of combining a good equation of state with two-phase predictive methods was proposed and applied by them. The whole work takes the reader into the useful utilization of the computer in designing and operating gas processes in all aspects.

The need for more accurate two-phase flow correlations demands solid theoretical investigation coupled with testing by data. The application of good physical and thermodynamic prediction methods might help in developing a future two-phase flow method.

CHAPTER III

PROGRAM GENERAL DESCRIPTION

The main purpose of this research was to develop a computer program that contained the more widely used two-phase flow calculation methods. The program was to incorporate good physical and thermodynamic properties predictive methods. This package was an equation of state which is capable of calculating thermodynamic and some physical properties, a viscosity correlation for both the liquid and vapor phases and a surface tension predictive method. The package contains five two-phase flow calculation procedures: one for upward vertical flow, another for both upward and downward vertical flow, a method for inclined flow, one for horizontal flow and one method is for all directions of flow. All of the methods, with the exception of those for vertical flow, have been modified to calculate all types of flow.

The Equation of State

The equation of state chosen for the prediction of thermodynamic and physical properties is the Soave-Redlich-Kwong equation of state. The package contained in the program is part of GPA*SIM (28).

Viscosity Correlation

Page 403

A subroutine was written based on the correlations of Thodos and co-workers as presented by Ried, Prausnitz and Sherwood (29). The subroutine is capable of calculating either the liquid viscosity or the vapor viscosity. It is interacting within the system and can be called at any point for the calculation of either liquid or vapor viscosity.

Surface Tension

The subroutine for surface tension is based on the equations presented in the GPSA Engineering Data Book 16-33 (30) and is capable of calculating the liquid-vapor surface tension of mixtures.

Two-Phase Flow Procedures

The five methods chosen for this work cover the three cases of application in the industry: vertical flow, inclined flow and horizontal flow. They were chosen from a wide field of methods available in the literature on the basis mentioned earlier. Other methods can be added to the package with a few minor alterations to the package.

The Duns and Ros Method (15)

This method is limited by the original authors to vertical-liquid and/or gas-liquid flow through a circular conduit from the bottom of the well to the well head. The method correlates the liquid hold-up and friction factor using four dimensionless numbers, which are:

$$RN = \text{gas velocity number} = V_{sg}\,(\rho_\ell g_c/g\sigma)^{1/4}$$

$$N = \text{liquid velocity number} = V_{sl}(\rho_\ell g_c/g\sigma)^{1/4}$$

$$N_d = \text{diameter number} = D(\rho_\ell g/\sigma)^{1/2}$$

$$N_\ell = \text{liquid viscosity number} = \mu_\ell(g/\rho_\ell\sigma^3 g_c)^{1/4}$$

The liquid hold-up E_ℓ is functionally related to the slip velocity, V_s, which is defined as follows:

$E_\ell V_{sg} + E_\ell V_{s\ell} - V_{s\ell}$

$$V_s = \frac{V_{sg}}{1-E_\ell} - \frac{V_{sl}}{E_\ell} = \frac{E_\ell V_{sg} - V_{s\ell}(1-E_\ell)}{E_\ell(1-E_\ell)} \qquad \text{III-1}$$

$E_\ell(1-E_\ell)\,(V_s) = (V_{s\ell} + V_{sg})E_\ell - V_{s\ell}$

The slip velocity is expressed in dimensionless form by:

$$S = V_s\ (\rho_\ell/g\sigma)^{1/4} \qquad \text{III-2}$$

The slip correlation S was obtained by correlating the governing group. Once S is obtained, V_s can be determined.

For Region I containing the bubble flow, plug flow and part of the froth flow regimes, S is covered by the following formulas:

$$S = F_1 + F_2N + F_3' \left(\frac{RN}{1+N}\right)^2 \qquad \text{III-3}$$

$$F_3' = F_3 - \frac{F_4}{N_d} \qquad \text{III-4}$$

F_1, F_2 and F_3 can be obtained from Figure 1. Region I extends from Zero N and RN up to the limit given by:

$$RN = L_1 + L_2N \qquad \text{III-5}$$

The factors L_1 and L_2 depend on the diameter number and are given in Figure 2. For Region II, which covers the slug flow regime and the remainder of the froth flow regime, the slip correlation is:

$$S = (1+F_5)\,\frac{(RN)^{.982} + F_6'}{(1 + F_7N)^2} \qquad \text{III-6}$$

where:

$$F_6' = .029\ N_d + F_6 \qquad \text{III-7}$$

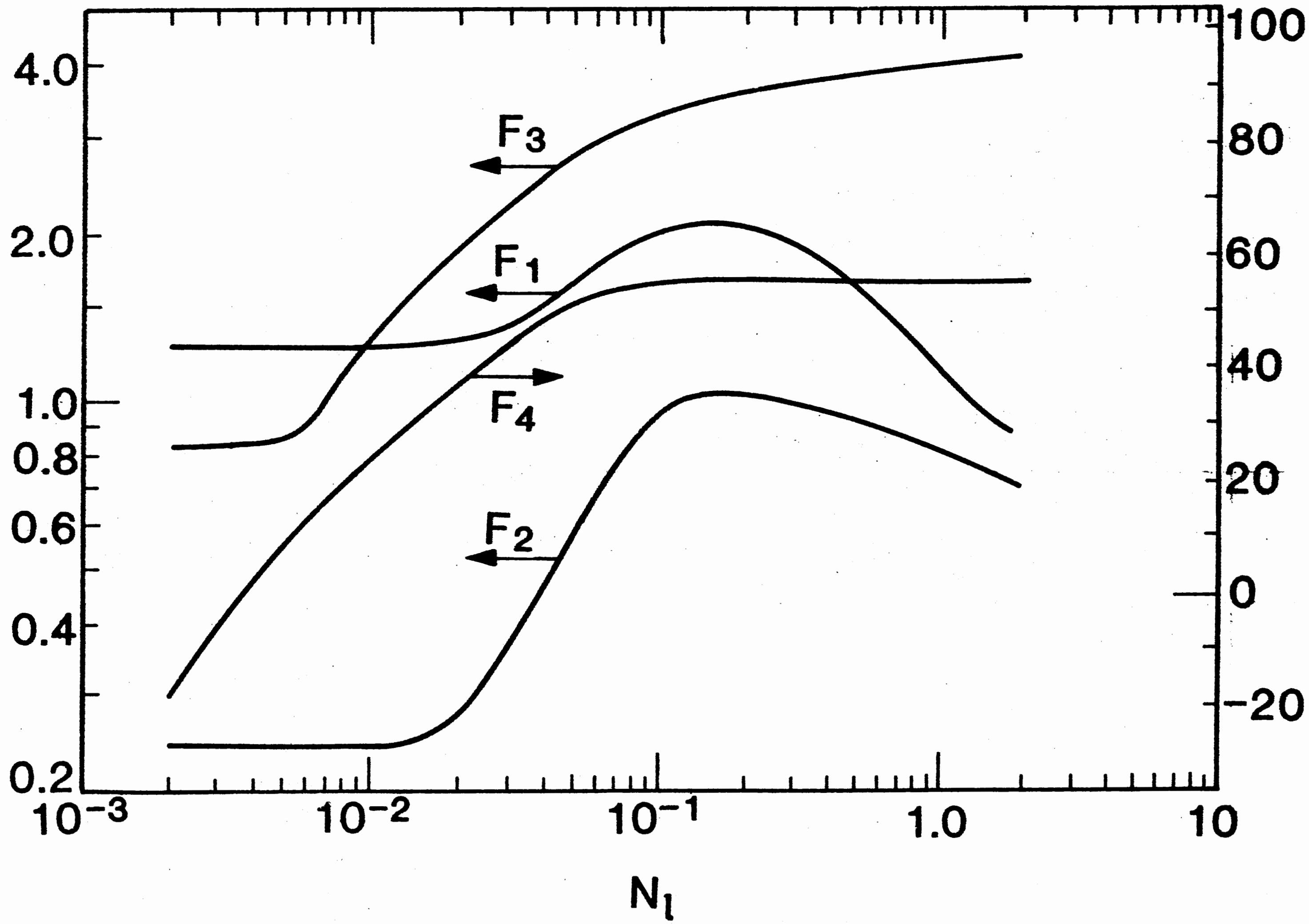

Figure 1. Non-dimensional Functions F_1, F_2, F_3 and F_4 Against Viscosity Number N_ℓ

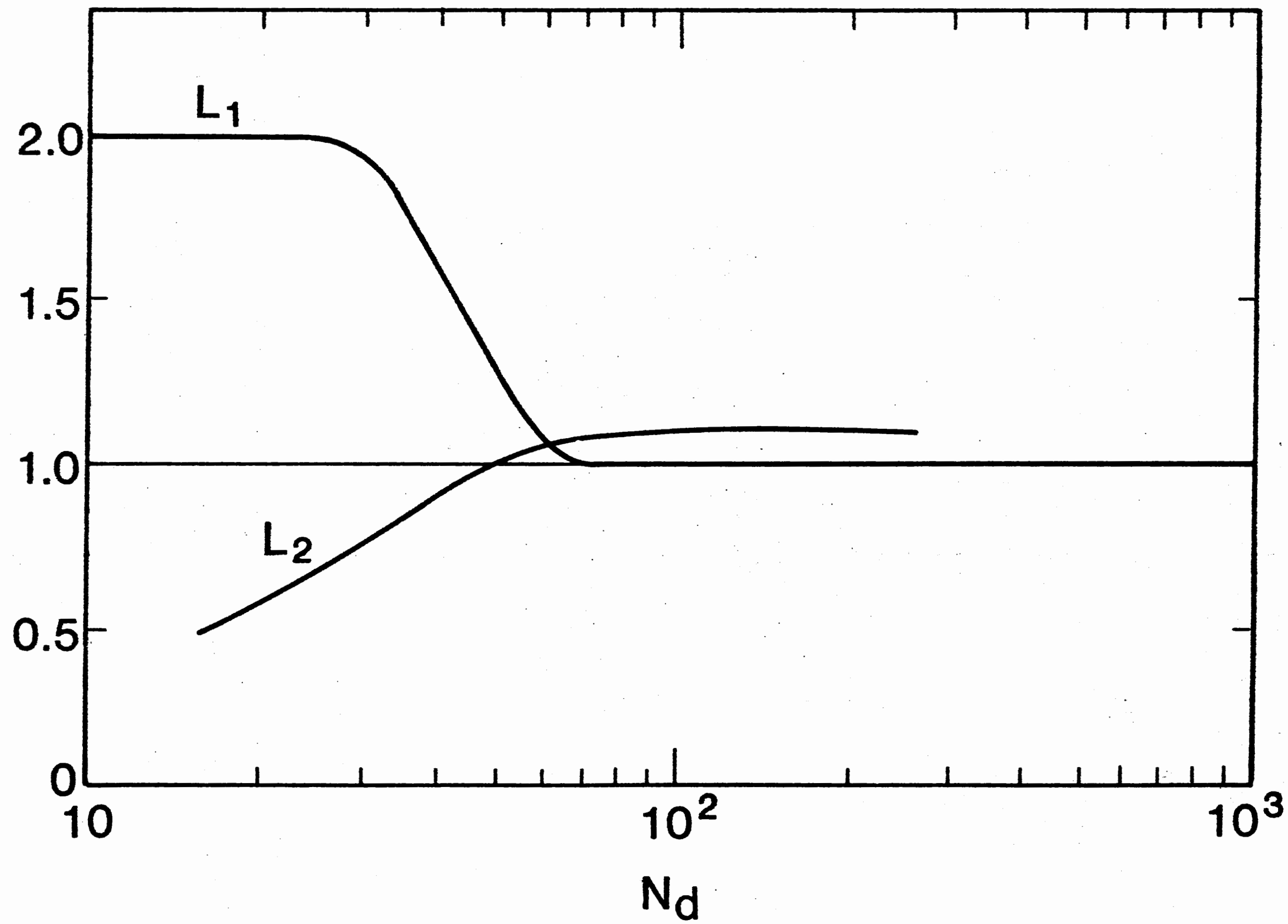

Figure 2. Non-dimensional Functions L_1 and L_2 Against Diameter Number N_d

Region II extends from the upper limit of Region I to the transition zone to mist flow given by:

$$RN = 50 + 36N \qquad \text{III-8}$$

As the gas velocity becomes very high, the liquid is then transported as small droplets (mist flow). There is virtually no slip between the gas and liquid droplets so V_s becomes zero. E_ℓ is obtained from the following equation:

$$E_\ell = \frac{1}{1 + V_{sg}/V_{s\ell}} \qquad \text{III-9}$$

For the transition zone extending from the upper limit of Region II to the limit given by:

$$RN \leqslant 75 + 84\, N^{.75} \qquad \text{III-10}$$

No hold-up correlation for the zone was presented. Instead, the total pressure gradient was approximated by linear interpolation, on the basis of the value of RN, between pressure gradient values obtained for the upper limit of Region I and the lower limit of Region III.

The static pressure gradient (G_{st}) is obtained by the following equation:

$$G_{st} = E_\ell + (1-E_\ell)\,\frac{\rho_g}{\rho_\ell} \qquad \text{III-11}$$

The frictional pressure gradient (G_{fr}) is the same for Regions I and II and is governed by the following equations:

$$G_{fr} = 2f_w\,\frac{N(N+RN)}{N_d} \qquad \text{III-12}$$

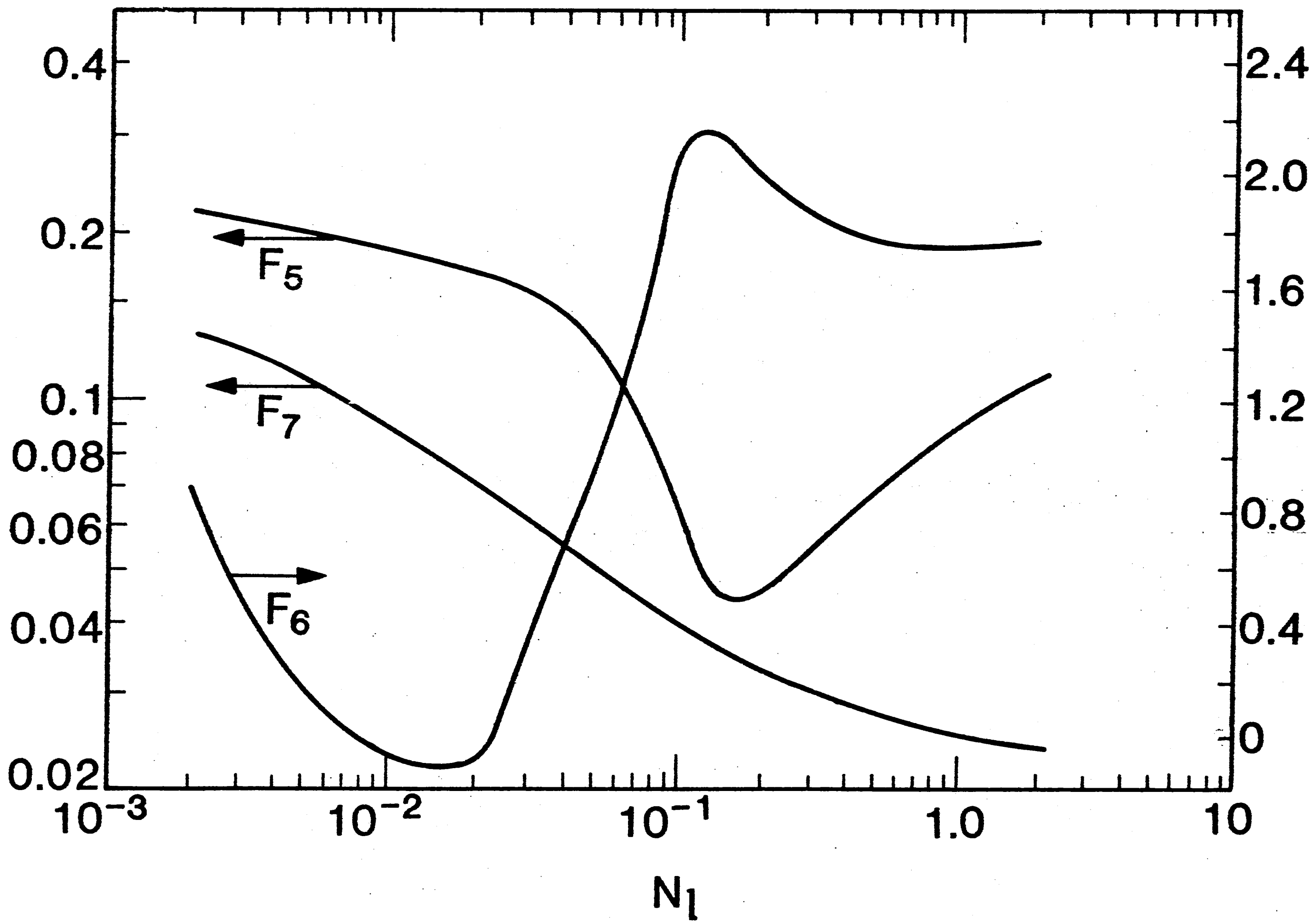

Figure 3. Non-dimensional Functions F_5, F_6, and F_7 Against Viscosity Number N_ℓ

$$f_w = f_1 \frac{f_2}{f_3} \qquad \text{III-13}$$

$$f_3 = f_1 \sqrt{V_{sg}/50v_{s\ell}}$$

The dimensionless factors f_1 and f_2 can be obtained from Figures 4 and 5 respectively.

In Region III the gas phase is continuous and friction originates from the drag of the gas on the pipe wall. Although slip is absent, there is a liquid film that covers the wall of the pipe and the normal roughness factor ε is no longer valid. Duns and Ros present a new ε which takes into account this added problem. Figure 6 allows the calculation of ε which can then be used to obtain f_1 from Figure 4. The frictional pressure gradient is obtained by equations III-14 and III-15.

$$f_w = f_1 \qquad \text{III-14}$$

$$G_{fr} = 2f_w \frac{\rho_{g(RN)}^{\ 2}}{\rho_\ell N_d} \qquad \text{III-15}$$

Duns and Ros obtained an equation that takes into account the contribution of acceleration in terms of the frictional and static pressure gradients.

$$G = \frac{G_{st} + G_{fr}}{1 - (\rho_\ell V_{s\ell} + \rho_g V_{sg})(V_{sg}/P)} \qquad \text{III-16}$$

One has to keep in mind that this total pressure gradient is dimensionless and needs to be multiplied by the appropriate density and acceleration values in order to obtain a numerical answer.

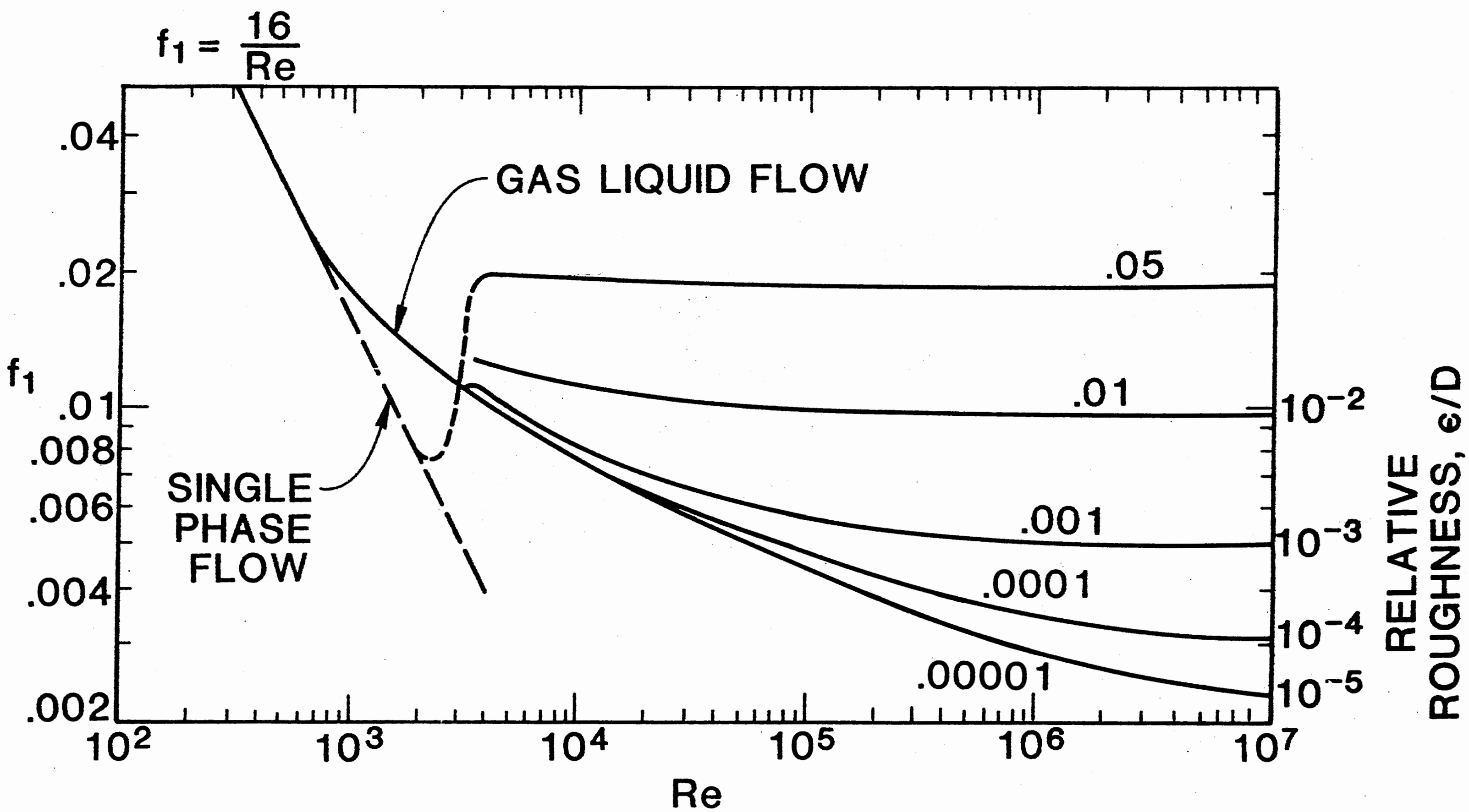

Figure 4. Non-dimensional Function f_1 Against Reynolds Number Re

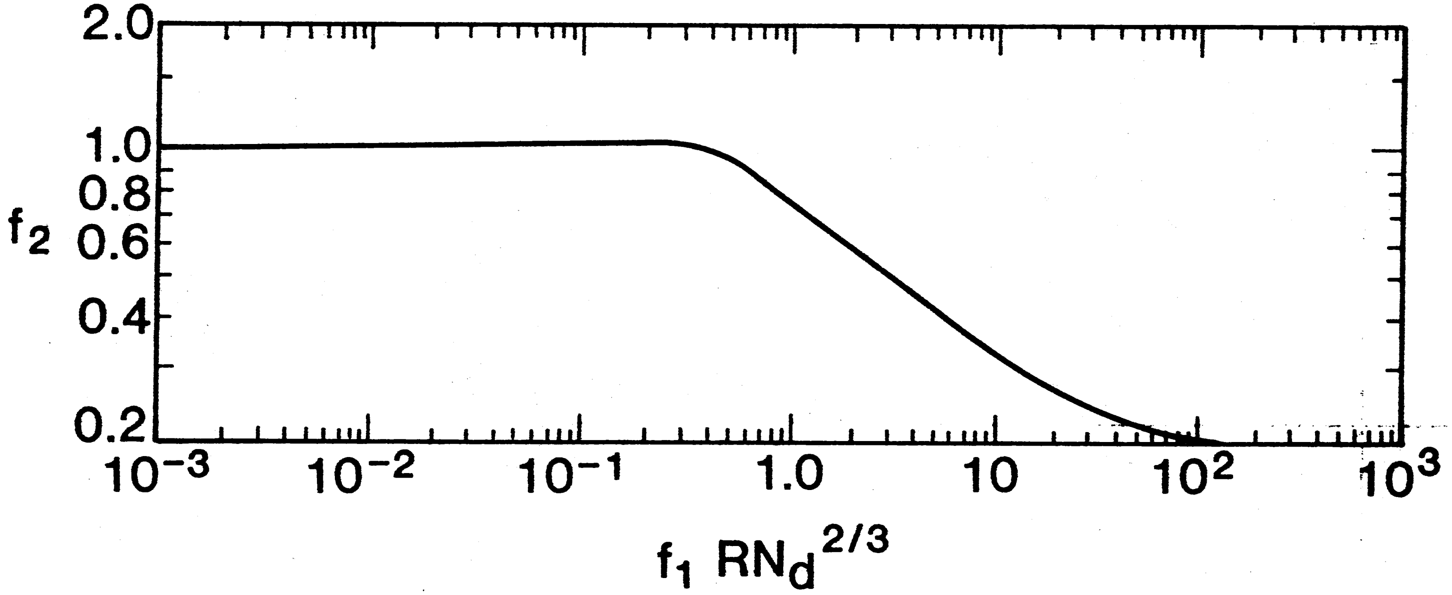

Figure 5. Non-dimensional Function F_2 Against $F_1 RN_d^{2/3}$

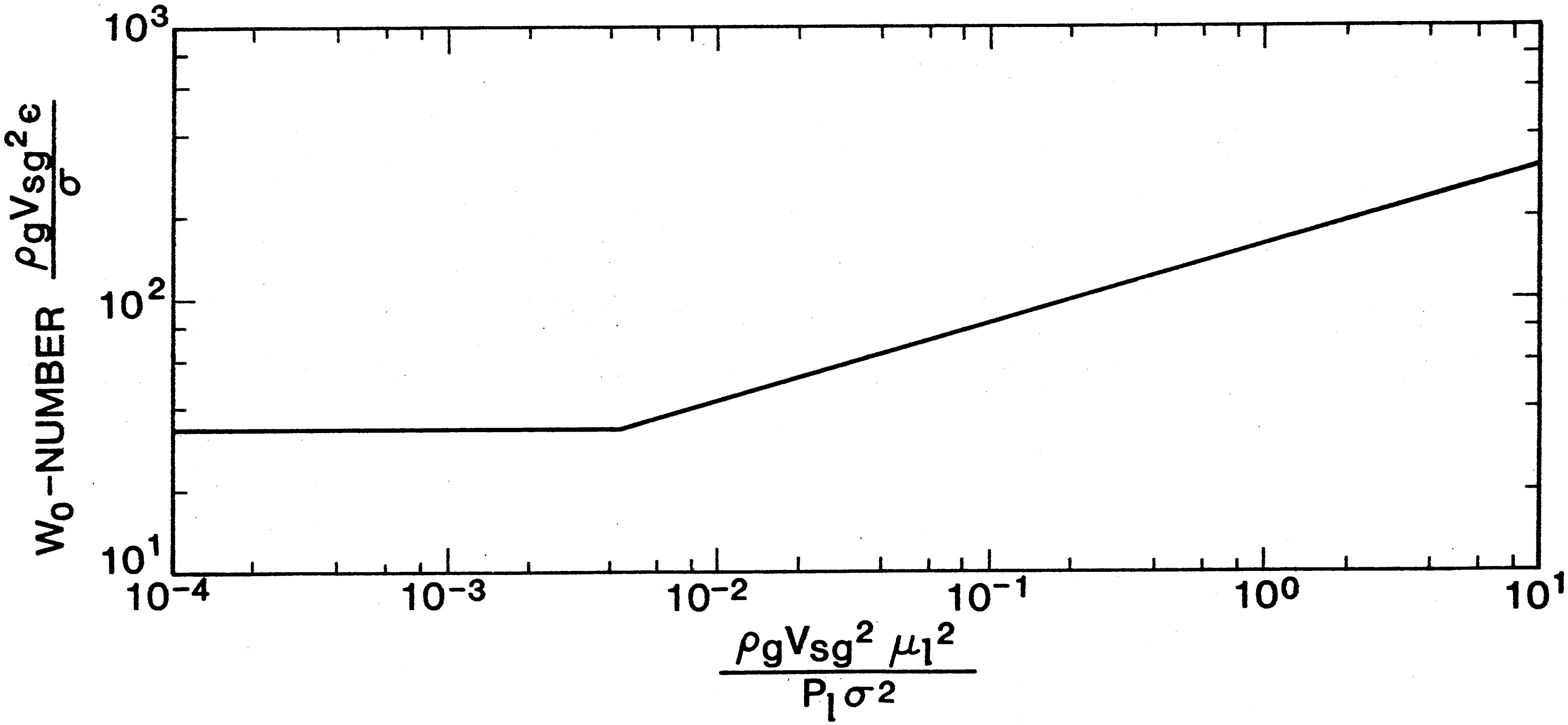

Figure 6. Correlation for the Film-Thickness ϵ Under Mist-Flow Conditions

The Orkiszewski Correlations (6,7,16)

This correlation is similar to that of Duns and Ross in its use of flow regime numbers to define the boundaries of flow regions. The flow regime parameters are:

$$\text{Bubble flow number } N_b = 1.071 - 0.2218\, V_{ns}{}^2/D \qquad \text{III-17}$$

$$\text{Slug flow number } N_s = 50 + 70\, V_{s\ell}(\rho_\ell/\sigma)^{1/4} \qquad \text{III-18}$$

$$\text{Mist flow number } N_m = 75 + 138\,[V_{s\ell}(\rho_\ell/\sigma)^{1/4}]^{.75} \qquad \text{III-19}$$

$$\text{Gas velocity number } N_{gv} = 1.938\, V_{sg}(\rho_\ell/\sigma)^{1/4} \qquad \text{III-20}$$

Bubble flow exists when the following inequality is satisfied:

$$V_{sg}/V_{ns} < N_b \qquad \text{III-21}$$

The liquid hold-up, two-phase density, Reynolds number and frictional pressure gradient are given by the following equations:

$$R_\ell = 0.5 - .625\, V_{ns} + [(.5 + .625\, V_{ns})^2 - 1.25\, V_{sg}]^{1/2} \qquad \text{III-22}$$

$$\rho_B = R_\ell \rho_\ell + (1-R_\ell)\rho_g \qquad \text{III-23}$$

$$Re_b = \rho_\ell D V_{s\ell}/\mu_\ell R_\ell \qquad \text{III-24}$$

$$\left(\frac{dP}{dZ}\right)_f = f_{tp}\rho_\ell (V_{s\ell}/R_\ell)^2/2g_c D \qquad \text{III-25}$$

The two-phase friction factor is obtained from the Colebrook equation (4) using the two-phase Reynolds number.

$$1/f_{tp}^{1/2} = -2 \log [(\varepsilon/3.7D) + 2.51/(Re_{tp} f_{tp}{}^{1/2})] \qquad \text{III-25a}$$

$$\text{Slug flow exists when } V_g/V_{ns} > N \text{ and } N_{gv} < N_s \qquad \text{III-26}$$

In this regime the Reynolds number is defined as:

$$Re_s = 1488\ \rho_\ell D V_{ns}/\mu_\ell \qquad \text{III-27}$$

Defining the quantities N_1 and N_2 as:

$$N_1 = 37200\ [-.35 + (0.1225 + 0.04931\ V_{ns}/D^{0.5})] \qquad \text{III-28}$$

$$N_2 = 37210\ [-.546 + (.2981 + 0.01849\ V_{ns}/D^{0.5})] \qquad \text{III-29}$$

V_r, the bubble rise velocity is given by:

.958

$$\text{if } Re_s > N_1 \quad V_r = (1.985 + 4.985 \times 10^{-5}\ R_{es})D^{0.5} \qquad \text{III-30}$$

$$\text{if } Re_s < N_2 \quad V_r = (3.097 + 4.958 \times 10^{-5}\ R_{es})D^{0.5} \qquad \text{III-31}$$

if $N_1 > Re_s > N_2$ then:

$$\gamma = (1.423 + 4.958 \times 10^{-5}\ R_{es})D^{0.5} \qquad \text{III-32}$$

$$V_r = 0.5\ [\gamma + (\gamma^2 + (13.59\mu_\ell)/\rho_\ell\ D^{0.5})^{.5}] \qquad \text{III-33}$$

The parameter Γ is calculated by:

$$\text{if } V_{ns} < 10 \quad \Gamma = [0.0127\ \log(\mu_\ell+1)]/D^{1.415} - 0.284 + 0.167\ \log V_{ns} + 0.113\ \log D \qquad \text{III-34}$$

$$\text{if } V_{ns} > 10 \quad \Gamma = [0.0274\ \log(\mu_\ell+1)]/D^{1.371} + 0.161 + 0.569\ \log D - \{[0.01\ \log(\mu_\ell+1)]/D^{1.571} + 0.397 - 0.63\ \log D\}\ \log V_{ns} \qquad \text{III-35}$$

Finally, the slug flow density and the frictional pressure drops are calculated:

$$\rho_s = (G_t + \rho_\ell V_r)/(V_{ns} + V_r) + \Gamma\rho_\ell \qquad \text{III-36}$$

and

$$(\frac{dP}{dZ})_f = (f_{tp}\rho_\ell V_{ns}{}^2/2g_cD)[(V_{s\ell}+V_r)/(V_{ns}+V_r) + \Gamma] \qquad \text{III-37}$$

The two-phase friction factor is obtained as for bubble flow.

Orkiszewski suggested the following averaging procedure for transition flow:

$$t_1 = (N_m-N_{gv})/(N_m-N_s) \qquad \text{III-38}$$

and

$$(\frac{dP}{dZ})_{trans.} = t_1 (\frac{dP}{dZ})_{slug} + (1-t_1)(\frac{dP}{dZ})_{mist} \qquad \text{III-39}$$

The transition flow exists when

$$N_m > N_{gv} > N_s \qquad \text{III-40}$$

The mist flow exists when

$$N_{gv} > N_m \qquad \text{III-41}$$

The correction term for relative roughness is applied where $10^{-3} < \varepsilon/D < .5$ defining:

$$N_w = (4.52 \times 10^{07})(V_{sg}\mu_\ell/\sigma)^2(\rho_g/\rho_\ell) \qquad \text{III-42}$$

if $N_w > .005$

$$\varepsilon/D = 4.14\sigma(N_w)^{.302}/(\rho_g V_{sg}{}^2 D) \qquad \text{III-43}$$

if $N_w < .005$

$$\varepsilon/D = 0.804\sigma/(\rho_g V_{sg}{}^2 D) \qquad \text{III-44}$$

and

$$R_{em} = 1488\ \rho_g D V_{sg}/\mu_g \qquad \text{III-45}$$

then

$$\left(\frac{dP}{dZ}\right)_f = f_{tp}\rho_g V_{sg}^{\ 2}/2g_c D, \qquad \text{III-46}$$

where f_{tp} is obtained from the Colebrook equations using ε/D and R_{em}.

The total pressure gradient is defined for all regions as follows:

$$\frac{dP}{dZ}_{total} = -\left[\left(\frac{dP}{dZ}\right)_{regeim} + \rho_{regeim}(g/g_c)\right] / (1-AC_{ns}) \qquad \text{III-47}$$

where

$$AC_{ns} = G_t V_{sg}/g_c \overline{P} \qquad \text{III-48}$$

The Beggs and Brill Correlations (18,19)

The Beggs and Brill correlations for liquid holdup and friction factor were developed using dimensionless variables. These variables are flow pattern dependent and are governed by the following equations:

$$\lambda = \text{volumetric liquid fraction} = V_{s\ell}/(V_{sg}+V_{s\ell}) = \frac{V_{s\ell}}{V_{ns}} \qquad \text{III-49}$$

$$X = \ell n(\lambda) \qquad \text{III-50}$$

where $V_{s\ell}$ = superficial liquid velocity
V_{sg} = " gas "

$$L_1 = \text{Exp}(-4.62 - 3.757X - 0.481\ X^2 - 0.027X^3) \qquad \text{III-51}$$

$$L_2 = \text{Exp}(1.061 - 4.602X - 1.609X^2 - 0.179X^3 + 0.635 \times 10^{-3}\ X^5) \qquad \text{III-52}$$

$$N_{FR} = \text{Froude number} = V_{ns}/gD \qquad \text{III-53}$$

$V_{ns} = V_{s\ell} + V_{sg}$

$$N = \text{liquid velocity number} = V_{s\ell}(\rho_\ell/g\sigma)^{0.25} \qquad \text{III-54}$$

$$RN = \text{gas velocity number} = V_{sg}(\rho_\ell/g\sigma)^{0.25} \qquad \text{III-55}$$

$$N_d = \text{diameter number} = D(\rho_\ell g/\sigma)^{.5} \qquad \text{III-56}$$

Three flow patterns were proposed:

1. If $N_{FR} < L_1$, the flow pattern is **segregated**
2. If $N_{FR} > L_1$ and $> L_2$, the flow pattern is **distributed**

3. If $L_1 < N_{FR} < L_2$, the flow pattern is intermittent.

The Beggs and Brill equations for liquid hold-up in the three flow regimes are shown in Table I. $H_L(0)$ is the liquid hold-up in a horizontal line. C+ (uphill) and C- (downhill) are the correction factors to be applied when the flow is uphill or downhill, respectively.

The hold-up at any angle is calculated from:

$$H_L(\theta) = H_L(0)\{1 + C[\sin(1.8\theta) - \frac{1}{3}\sin^3(1.8\theta)]\} \qquad \text{III-64}$$

provided that

$$H_L(0) \geq \lambda \qquad \text{III-65}$$

and

$$H_L(\theta) \geq 0 \qquad \text{III-66}$$

The two-phase friction factor is calculated as follows:

$$\frac{f_{tp}}{f_{ns}} = e^{S_B} \qquad \text{III-67}$$

where

$$S_B = [\ell n(y)]/\{-0.0523 + 3.182\ell n(y) - 0.8725[\ell n(y)]^2 + 0.01853[\ell n(y)]^4\} \qquad \text{III-68}$$

and

$$y = \frac{\lambda}{[H_L(\theta)]^2} \qquad \text{III-69}$$

The non-slip friction factor (f_{ns}) is obtained from the Moody diagram using

$$Re_{ns} = \frac{G_t D}{\mu_\ell \lambda + \mu_g(1-\lambda)} = \qquad \text{III-70}$$

TABLE I

EQUATIONS FOR PREDICTING LIQUID HOLDUP

Horizontal Flow Pattern	Horizontal Holdup	(C+) Uphill	(C−) Downhill
Segregated	$H_L(0) = \frac{0.98\lambda^{0.4846}}{N_{FR}^{0.0868}}$ III-57	$C+ = (1-\lambda)\ell n \left[\frac{0.011\ N_\ell^{3.539}}{\lambda^{3.768} N_{FR}^{1.614}}\right]$ III-60	$C- = (1-\lambda)\ell n \left[\frac{4.7\ N_\ell^{0.1244}}{\lambda^{0.3692} N_{FR}^{0.5056}}\right]$ III-63
Intermittent	$H_L(0) = \frac{0.845\lambda^{0.5351}}{N_{FR}^{0.0173}}$ III-58	$C+ = (1-\lambda)\ell n \frac{2.96\lambda^{0.305} N_{FR}^{0.0978}}{N_\ell^{0.4473}}$ III-61	Same as Segregated III-63
Distributed	$H_L(0) = \frac{1.065\lambda^{0.5824}}{N_{FR}^{0.0600}}$ III-59	$C+ = 0$ III-62	Same as Segregated III-63

Equation III-67 becomes unbounded in the interval $1 < y < 1.2$. For that interval S_B is calculated as:

$$S_B = \ln(2.2y - 1.2) \qquad \text{III-71}$$

The total pressure gradient is then calculated from

$$\frac{-dP}{dZ} = \frac{\frac{g}{g_L} \sin\theta[\rho_\ell H_L + \rho_g(1-H_L)] + \frac{f_{tp} G_t V_{ns}}{2g_c D}}{1 - \frac{[\rho_\ell H_L + \rho_g(1-H_L)]V_{ns} V_{sg}}{g_c P}} \qquad \text{III-72}$$

<u>The American Gas Association-American</u>

<u>Petroleum Institute Method (12,13)</u>

In this method, henceforth referred to as the AGA method, the pressure drop due to friction is calculated by:

$$f_{tp}/f_o = 1 - \frac{\ln\lambda}{S} \qquad \text{III-73}$$

where

$$S = 1.281 - 0.478(-\ln\lambda) + 0.444(-\ln\lambda)^2 - 0.094\ (-\ln\lambda)^3 + 0.00843(-\ln\lambda)^4 \qquad \text{III-74}$$

and

$$f_o = 0.0014 + \frac{0.125}{Re_{tp}0.32} \qquad \text{III-75}$$

The calculation of Re_{tp} (the two-phase Reynolds number) involves a trial and error procedure consisting of the following steps:

1. Estimate a value of $\overline{R}_L$ (the liquid hold-up)
2. Determine the value of λ from III-49
3. Calculate an approximate value of Re_{tp}.

$$Re_{tp} = \frac{DV_{ns}\rho_{tp}}{\mu_{tp}} \quad \text{III-76}$$

where

$$\rho_{tp} = \rho_{\ell} \frac{\lambda^2}{\overline{R}_L} + \rho_g \frac{(1-\lambda)^2}{(1-\overline{R}_L)} \quad \text{III-77}$$

and

$$\mu_{tp} = \mu_{\ell}\lambda + \mu_g(1-\lambda) \quad \text{III-78}$$

4. Using the value of λ and calculated value of Re_{tp}, obtain a value of $\overline{R}_L$ from Figure 7. If the value of $\overline{R}_L$ agrees with the assumed value within 5%, the calculation is satisfactory. If not, repeat steps 1 through 3 using the new value of $\overline{R}_L$.

The frictional pressure drop is calculated by:

$$\Delta P_F = \frac{2\ f_{tp}\ L\ V_{ns}^{2}\ \rho_{tp}}{144\ g_c D} \quad \text{III-79}$$

The pressure drop due to elevation changes is calculated using the Flanigan correlation (11) and the superficial gas velocity (V_{sg}):

$$\phi = \frac{1}{1 + 0.3264\ V_{sg}^{\ 1.006}} \quad \text{III-80}$$

and

$$\Delta P_F = \frac{\phi \rho_{\ell} \Sigma H}{144} \quad \text{III-81}$$

where

ΣH = sum of elevation changes.

The pressure drop due to acceleration is calculated from:

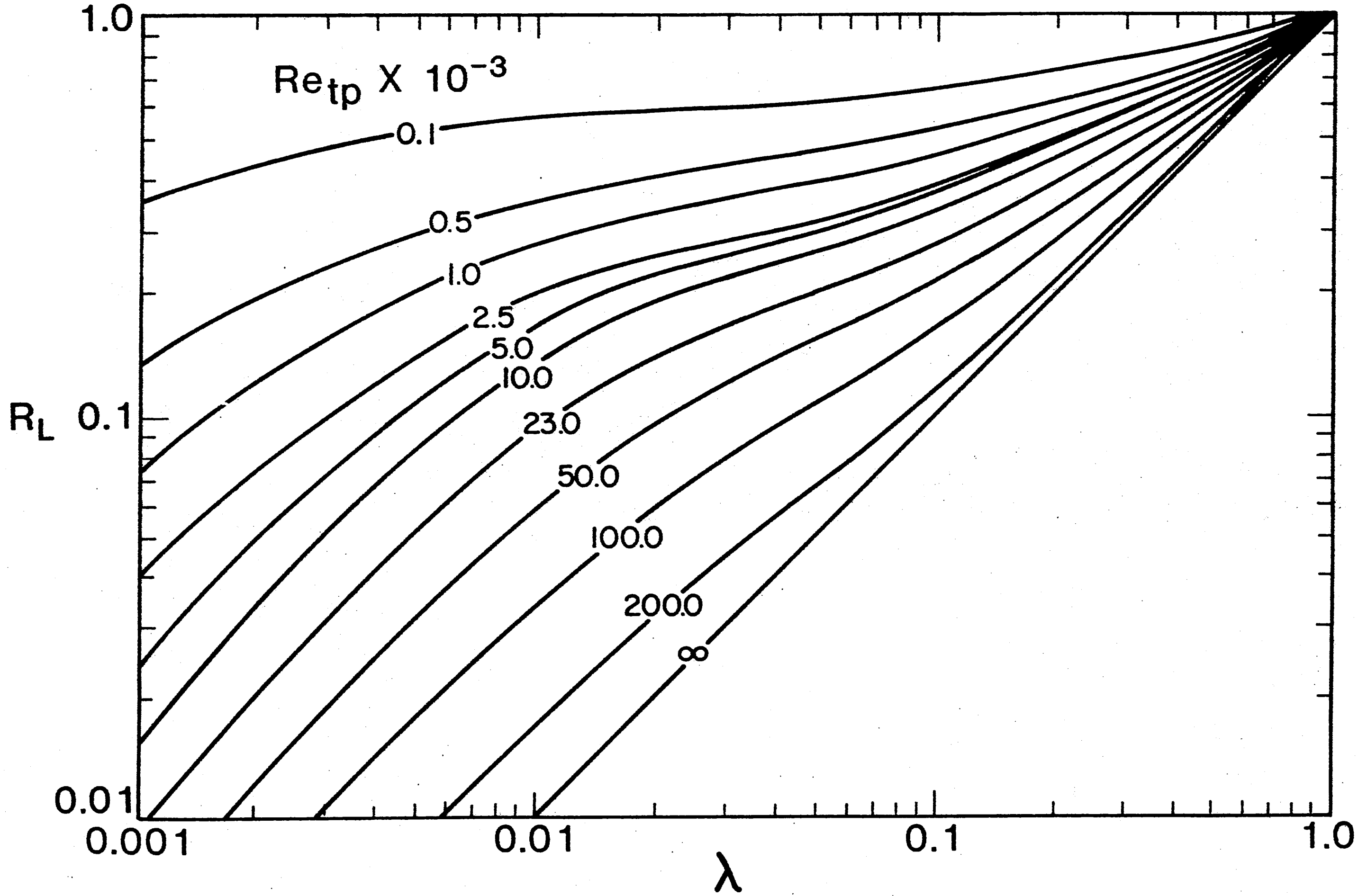

Figure 7. Liquid Holdup Correlation for Horizontal Pipe

$$\Delta P_A = \frac{1}{144 g_c}\left\{\left[\frac{\rho_g V_{SG}^2}{(1-R_L)} + \frac{\rho_\ell V_{S1}^2}{R_L}\right]_{\text{downstream}} - \left[\frac{\rho_g V_{sg}^2}{(1-R_L)} + \frac{\rho_\ell V_{S1}^2}{R_L}\right]_{\text{upstream}} \cos\theta\right\} \qquad \text{III-82}$$

where

θ = the angle of the pipe bend.

The total pressure drop is the sum of the pressure drops due to friction, elevation changes and acceleration.

$$\Delta P_{total} = \Delta P_F + \Delta P_E + \Delta P_A \qquad \text{III-83}$$

The Lockhart-Martinelli Correlations (4,9)

The Lockhart-Martinelli correlating parameter is

$$X = [(dP/dZ)_\ell \;/(dP/dZ)_g]^{1/2} \qquad \text{III-84}$$

where the pressure gradient terms are those which would result if each phase occupied the entire conduit separately. The correlating parameter is then used to find a multiplying factor that can be used to obtain the two-phase frictional pressure drop from that calculated for single-phase flow:

$$\phi_\ell^2 = \left(\frac{dP}{dZ}\right)_{tp} \Big/ \left(\frac{dP}{dZ}\right)_\ell \qquad \text{III-85}$$

The function ϕ_ℓ was presented in graphical form. The flow mechanism was chosen to divide the flow into four types:

1. Turbulent-turbulent flow - if the Reynolds numbers for both the liquid and vapor phases is greater than 2000, the corresponding ϕ_ℓ is $\phi_{\ell,tt}$.

2. Turbulent-viscos flow - if the Reynolds number for the liquid phase is greater than 2000 and the Reynolds number for the vapor phase is less than 2000, the corresponding ϕ_ℓ is $\phi_{\ell,tv}$.

3. Viscous-turbulent flow - if the Reynolds number for the liquid phase is less than 2000 and the Reynolds number for the vapor phase is greater than 2000, the corresponding ϕ_ℓ is $\phi_{\ell,vt}$.

4. Viscous-viscous flow - if both Reynolds numbers are less than 2000, the corresponding ϕ_ℓ is $\phi_{\ell,vv}$.

DeGance and Atherton (4) curve-fitted the correlations in the following form:

$$\phi_\ell = \mathrm{EXP}\left\{\sum_{i=1}^{n} a_i (\ell n X)^{i=1}\right\} \qquad \text{III-86}$$

The coefficients for the fit are presented in Table II. Once the type of flow for each phase is known and the parameter X is obtained, the two-phase frictional pressure drop is then calculated by using equations III-85 and III-86.

Pressure Drop Calculations

An iterative procedure is used to determine the pressure at the end of the pipe. The procedure is as follows:

1. The pipeline is divided into an appropriate number of segments.

2. The outlet temperature and pressure are estimated for the end of the segment.

3. A flash calculation with the average $\frac{T_1+T_2}{2}$ temperature and pressure $\frac{P_1+P}{2}$

of the segment is performed. Physical and thermodynamic properties are obtained.

4. The pressure drop for the segment is calculated using one of the two-phase pressure drop methods.

5. If the pressure at the end of the segment is acceptable within a tolerance, proceed to Step 6. If not, Steps 3 and 4 are repeated using the calculated pressure.

6. An energy balance is performed as follows:

$$Q = UA_p(T_S - T_A) \quad = \Delta H$$

$$H_{out} = H_{in} + Q \quad = H_{in} + \Delta H \quad , \quad \Delta H = H_2 - H_1$$

where:

Q = amount of heat transferred to the pipe segment

U = the overall heat transfer coefficient for the segment

T_S = temperature of the surroundings

T_A = average temperature for the segment

$H_{out,in}$ = total enthalpy of the fluid

7. A flash calculation is performed for the condition at the end of the segment. The temperature at the end of the segment is obtained.

8. If the temperature is the same as in Step 2, the calculations for the next pipline segment are performed. If not, Steps 2 through 7 are repeated.

The use of the equations of state in this manner would automatically account for temperature changes resulting from fluid expansion (Toule-Thompson effect). Another advantage of the use of the equation of state is in the case of vertical flow. Since the static pressure heat is of great importance in the total pressure drop calculations, an accurate estimation of the relative amounts of liquid and vapor is critical. The equation of state provides such an estimation with accuracy, thus allowing the determination of the total pressure drop.

TABLE II

CURVE FITS FOR MARTINELLI FUNCTIONS

	ϕ_{ℓ},vv	ϕ_{ℓ},tv	ϕ_{ℓ},tt	ϕ_{ℓ},vt	Re,1	Re,2
a_1	0.97995	1.24907	1.44065	1.23807	-0.25522	-0.25522
a_2	-0.42951	-0.44314	-0.50445	-0.46844	-0.10583	-0.10573
a_3	0.09563	0.06680	0.06212	0.07189	-0.02893	-0.02893
a_4	-0.00547	-0.00521	-0.00106	-0.00444	-0.00884	-0.00884
a_5	0.00142	-0.00057	-0.00101	-0.00070	-	-
a_6	0.00011	0.00012	0.00003	0.00012	-	-
a_7	-	-	0.00002	-	-	-

CHAPTER IV

TEST CASES

Five test cases were chosen to demonstrate the capabilities of the program, to compare the methods used and to study the effect of different parameters on two-phase pressure drop. In addition, the cases were intended to cover as wide a range as possible of gas and liquid flow ratios and pipeline elevation profiles.

The five cases are:

Case 1 verTical

As originally presented by Gould (31), the pipeline is 30 miles (48.3 km) long with a uniform rise of 50 ft/mile (9.47 m/km). The inside pipe diameter is 15 inches (38.1 cm). Inlet conditions were set at 915 psia (6.31 MPa) and 140^{o}F (60^{o}C) with an equivalent gas flow rate of 100 MMSCF/D. Two overall heat transfer coefficients and two relative roughness factors were used. The composition of the fluid and the specifications for the case are shown in Table III.

Case 2 veTical

The case was based on the information provided for well 22 by Orkiszwski (17). Since the original article did not provide the composition of the fluid, an attempt was made to create a composition that matched the values for density and overall API gravity stated

TABLE III

SPECIFICATIONS FOR CASE 1

Inclined Gas Pipeline

Composition:

Component	Mole Fraction
C_1	76.432
C_2	7.923
C_3	4.301
C_4	3.060
C_5	1.718
C_6	1.405
C_7	2.992
N_2	1.375
CO_2	0.794

Pipeline Conditions:

Equivalent Gas Flow Rate	100 MMSCFD
Diameter of Pipe	15 inches
Total Length	30 miles
Elevation Change	50 feet per mile
Roughness ε/D	4×10^{-4}
	0.0
Overall Heat Transfer Coefficient U	0.5 Btu/hrft^{2}F
	1.0 Btu/hrft^{2}F
Pressure	915 psia
Temperature	140°F
Temperature of the Surroundings	50°F

in the article. The specifications for the pipeline are shown in Table IV.

Case 3

In this case, an attempt was made to create a fluid that had a high liquid content in order to study the effect of pipeline diameter on temperature and pressure profiles and change in diameter on volumetric liquid fraction in the pipe. Table V contains the composition of the fluid and the pipeline specifications.

Case 4 Horizontal

The information for Case 4 was provided on a confidential basis (32). The pipeline connects an offshore platform to a processing plant on shore. The inside diameter of the pipeline is 19 inches (48.26 cm). The pipeline is 64.91 miles (104.505 km) long with a total rise of 249 ft (75.9m). Two relative roughness factors were chosen. The overall heat transfer coefficient was chosen in order to match the known temperature profile for the line. The composition and specifications are in Table VI.

Case 5

This is an artificial example that is based on Case 1. Elevation changes were dropped, but all other conditions (with the exception of one overall heat transfer coefficient) remained the same. The purpose of Case 5 was to study the effect of elevation change on the conditions studied in Case 1. Table VII contains the specifications for Case 5.

TABLE IV

SEPCIFICATIONS FOR CASE 2

Vertical Upward Flow Pipeline	
Composition:	
Component	Mole Fraction
C_1	40.1
C_2	26.5
C_3	0.2
Heavy Component	33.2
Heavy Component Specification:	
Normal Boiling Point	600°F
API Gravity	29
Molecular Weight	249
Pipeline Conditions:	
Equivalent Gas Flow Rate	1.58 MMSCFD
Diameter of Pipe	2.988 inches
Total Length	.74 miles
Elevation Changes	3924 feet (.74 miles)
Roughness ε/D	.00241
Overall Heat Transfer Coefficient U	1.0 $Btu/hrft^2F$
	1.5 $Btu/hrft^2F$
Pressure	1500 psia
Temperature	150°F
Temperature of the Surroundings	100°F

TABLE V

SPECIFICATIONS FOR CASE 3

Horizontal Flow Pipeline

Composition:

Component	Mole Fraction
C_1	30.1
C_2	26.5
C_3	0.2
Heavy Component	43.2

Heavy Component Specification:

Normal Boiling Point	$600^{\circ}F$
API Gravity	29
Molecular Weight	249

Pipe Line Conditions:

Equivalent Gas Flow Rate	15 MMSCFD
Diameter of Pipe	10 inches
	8 inches
	6 inches
Total Length	8 miles
Elevation Changes	0 feet
Roughness ε/D	6×10^{-4}
Overall Heat Transfer Coefficient U	0.5 Btu/hrft^{2}F
Pressure	1000 psia
Temperature	$127^{\circ}F$
Temperature of the Surroundings Ambient Tep	$70^{\circ}F$

TABLE VI

SPECIFICATIONS FOR CASE 4

Simulated Pipeline	Horizontal

Composition:

ID	Component	Mole Fraction
46	N_2	.58
49	CO_2	2.11
50	H_2S	0.01
2	C_1	79.79
3	C_2	7.06
4	C_3	4.82
5	$i\text{-}C_4$	.94
6	$n\text{-}C_4$	1.64
7	$i\text{-}C_5$	.61
8	$n\text{-}C_5$	.62
10	$n\text{-}C_6$	.55
11	$n\text{-}C_7$	.57
12	$n\text{-}C_8$	.29
13	$n\text{-}C_9$	.15
14	$n\text{-}C_{10}$	.26

Pipeline Conditions:

Equivalent Gas Flow Rate	155 MMSCFD
Diameter of Pipe	19 inches
Total Length	64.91 miles
Elevation Changes	249 94 feet = 0.05 mile
Roughness ε/D	1.26×10^{-4}
Overall Heat Transfer Coefficient	1.0 Btu/hrft^{2}F
Pressure	1379 psia
Temperature	129°F
Temperature of the Surroundings	50°F

TABLE VII

SPECIFICATIONS FOR CASE 5

Horizontal Pipeline
(Gas Flow)

Composition:

6 AH
7 L-H
8 BB
10 AH

Component		Mole Fraction	
C_1	1- 2,	76.432	2
C_2	2- 3,	7.923	3
C_3	3- 4,	4.301	4
C_4	4- 6	3.060	
C_5	5- 5,	1.718	
C_6	6 - 10,	1.405	
C_8	7- 2,	2.992	
N_2	8- 16,	1.375	
CO_2	9, 49,	.794	

Pipeline Conditions:

Equivalent Gas Flow Rate	100 MMSCFD
Diameter of Pipe	15 inches
Total Length	30 miles
Elevation Change	0 feet
Roughness ϵ/D	4×10^{-4}
	0.0
Overall Heat Transfer Coefficient U	0.1 Btu/hrft^{2}F
	0.5 Btu/hrft^{2}F
Pressure	915 psia
Temperature	140^{o}F
Temperature of the Surroundings	50^{o}F

CHAPTER V

DISCUSSION OF RESULTS

For the five test cases presented in Chapter IV, pressure drops were calculated using the applicable methods. The numerical results are presented in Appendices A, B, C, D, and E. Figures 8 through 26 represent the results as functions of the distance or the length of the pipeline. Each distance was divided into a number of segments. The segment length was chosen such that an optimum number of calculations can be conducted. An increase in the number of segments above the optimum number had little effect on the results.

In this chapter the results of each case will be discussed separately unless the cases are similar in nature.

The fluid in Case 1 is mostly gas flowing in a large pipe (I.D. 15 inches) with a slight rise (50 ft/mile). The methods used to calculate the pressure drops were the AGA, Beggs and Brill, and Lockhart and Martinelli. The most striking result is the effect of relative roughness on pressure drop. In Figures 8 and 9 the change from a smooth pipe ($\varepsilon = 1.0 \times 10^{-10}$) to a rough one ($\varepsilon = 5. \times 10^{-4}$ resulted in a large increase in pressure drop. Although an increase was expected, its magnitude in the case of Lockhart and Martinelli is disturbing. Several reasons that contribute to this are:

1. The Lockhart-Martinelli correlations were based on the air-water system flowing through smooth, small diameter pipes (.06-1.0

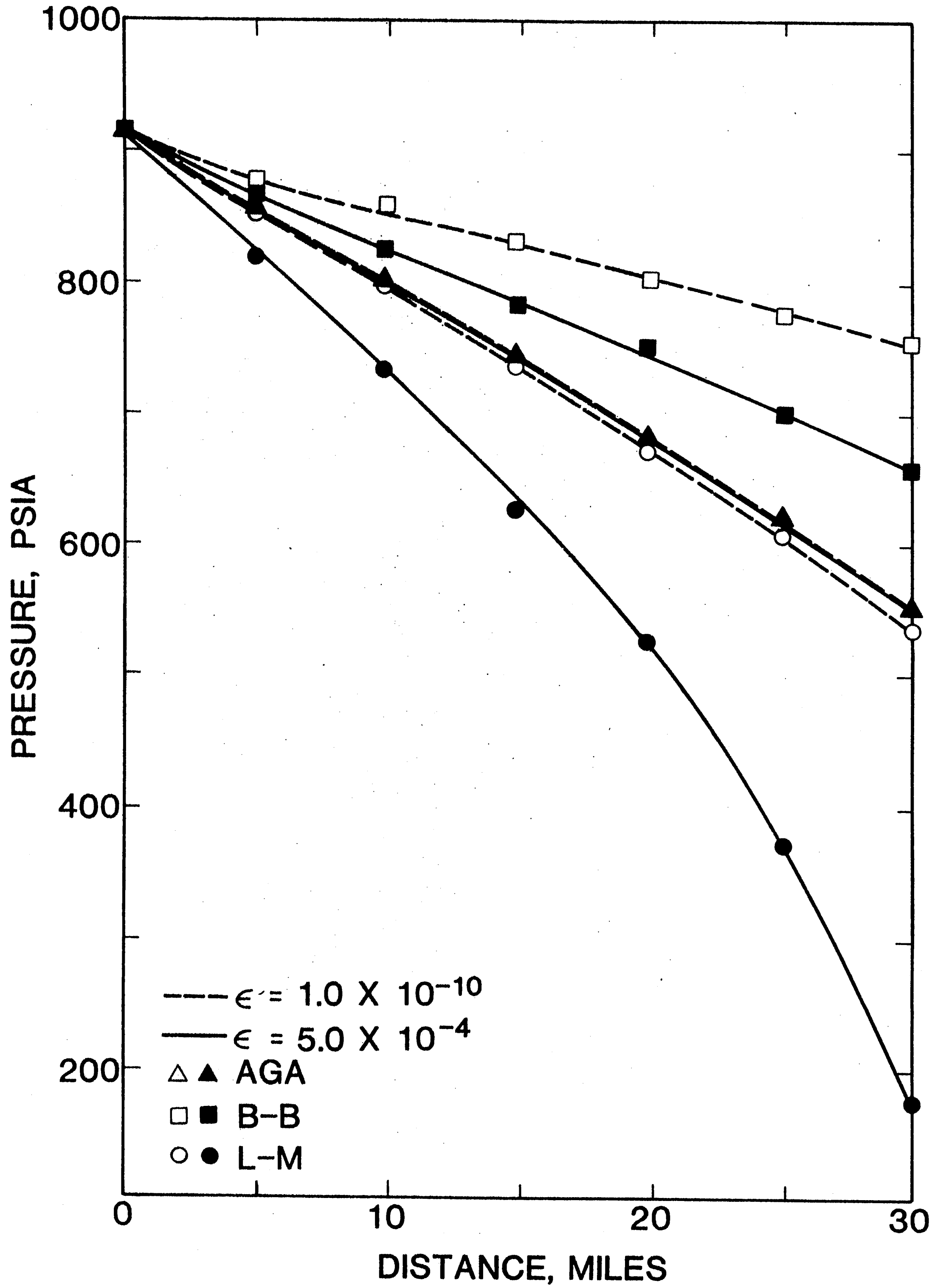

Figure 8. Pressure Profile for Case 1
(U = 1.0 Btu/hrft^{2}F)

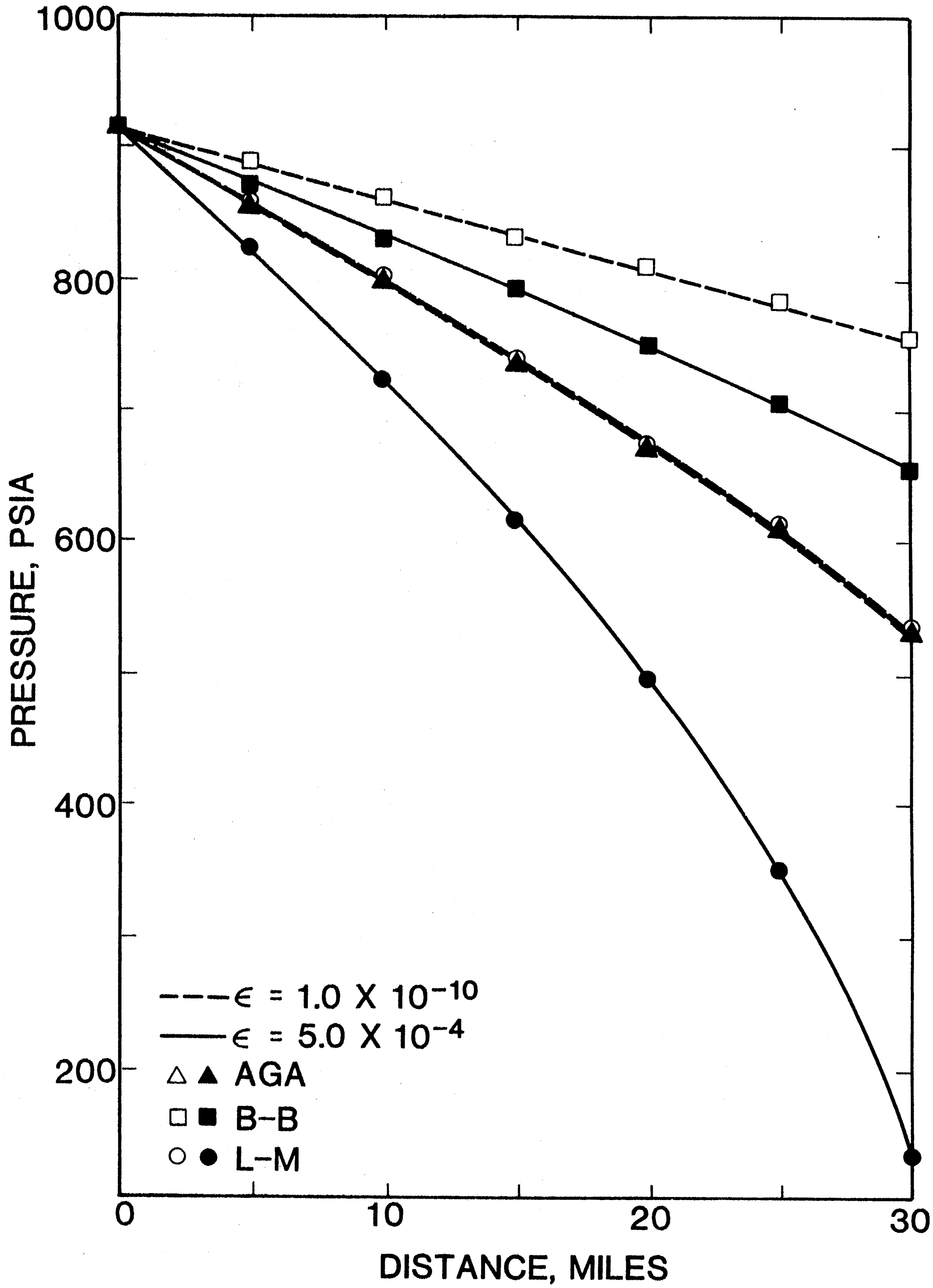

Figure 9. Pressure Profile for Case 1
($U = 0.5$ Btu/hrft^{2}F)

inches). Since the correlations are purely empirical, the use of such correlations in calculating pressure drops in large diameter, rough pipes will result in an error in calculation.

In support of this, one should note that the pressure drops in a smooth pipe for the AGA and the Lockhart-Martinelli methods are practically the same. The AGA was programmed for smooth pipes only, since their holdup correlation was based on data obtained from such pipes. Gangriwala (33) obtained similar results in spite of the fact that he used a different fluid.

2. Several authors (34,35,36) have argued that since the Lockhart-Martinelli correlations were based on an air-water system at one atmosphere they work well if the fluid used is the same or if the gas density equals that of air at atmospheric pressure.

On the other hand, the Beggs and Brill correlation predicted a lower pressure drop than the other two methods, even for a rough pipe. Gregory (36) states that "the effect of the angle of inclination of the pipe is small for angles up to 10°, measured from the horizontal. There does not appear to be a significant improvement in the prediction accuracy resulting from the use of the Beggs and Brill inclination correction factor in this range of angles."

The angle for case 1 is 0.54°. The correlation was developed for use in inclined flow. Such a small angle of inclination probably causes the conservate prediction.

The effect of the overall heat transfer coefficient (U) on pressure drop seems to be negligible if Figures 8 and 9 are compared. The effect on the temperature, as expected, is more noticeable. When a lower U is used less heat is transferred to the surroundings.

Temperature drop is therefore lowered as illustrated in Figures 10 and 11.

The effect of the over all heat transfer coefficient is more noticeable in the volumetric liquid fractions, as can be seen in Figures 12 and 13. The fraction of pipe volume occupied by the liquid is greater for all methods when the value of U is large. The fraction actually increases in spite of pressure loss. This is due to the rapid cooling at a nearly constant pressure drop in the first miles. As the amount of liquid increases the pressure drop increases, resulting in vaporization of some of the liquid.

Case 2 involves upward vertical flow of a fluid with a high liquid content. The two methods for purely vertical flow (Duns and Ros, Orkiszewski) predict similar pressure drops, while the AGA and Beggs and Brill methods predict slightly lower and higher pressure drops respectively.

No lines were drawn in Figure 15 in order not to crowd the graph. The overall heat transfer coefficient had a slight effect on both the temperature profile and the pressure drop. This is a clue to the facts that the pipeline was relatively short (.74 miles) and that the temperature of the surrounding was relatively high (100^{o}F).

Case 3 was intended to illustrate the effect of pipe diameter on pressure drops. As expected, larger diameter pipes cause lower pressure drops. The methods worked in a manner similar to their performance in Case 1, with Beggs and Brill predicting the lowest pressure drop and Lockhart and Martinelli predicting the highest. For the 6" diameter pipeline both methods failed to converge. Lockhart and Martinelli predicted an outlet pressure below zero, while Beggs

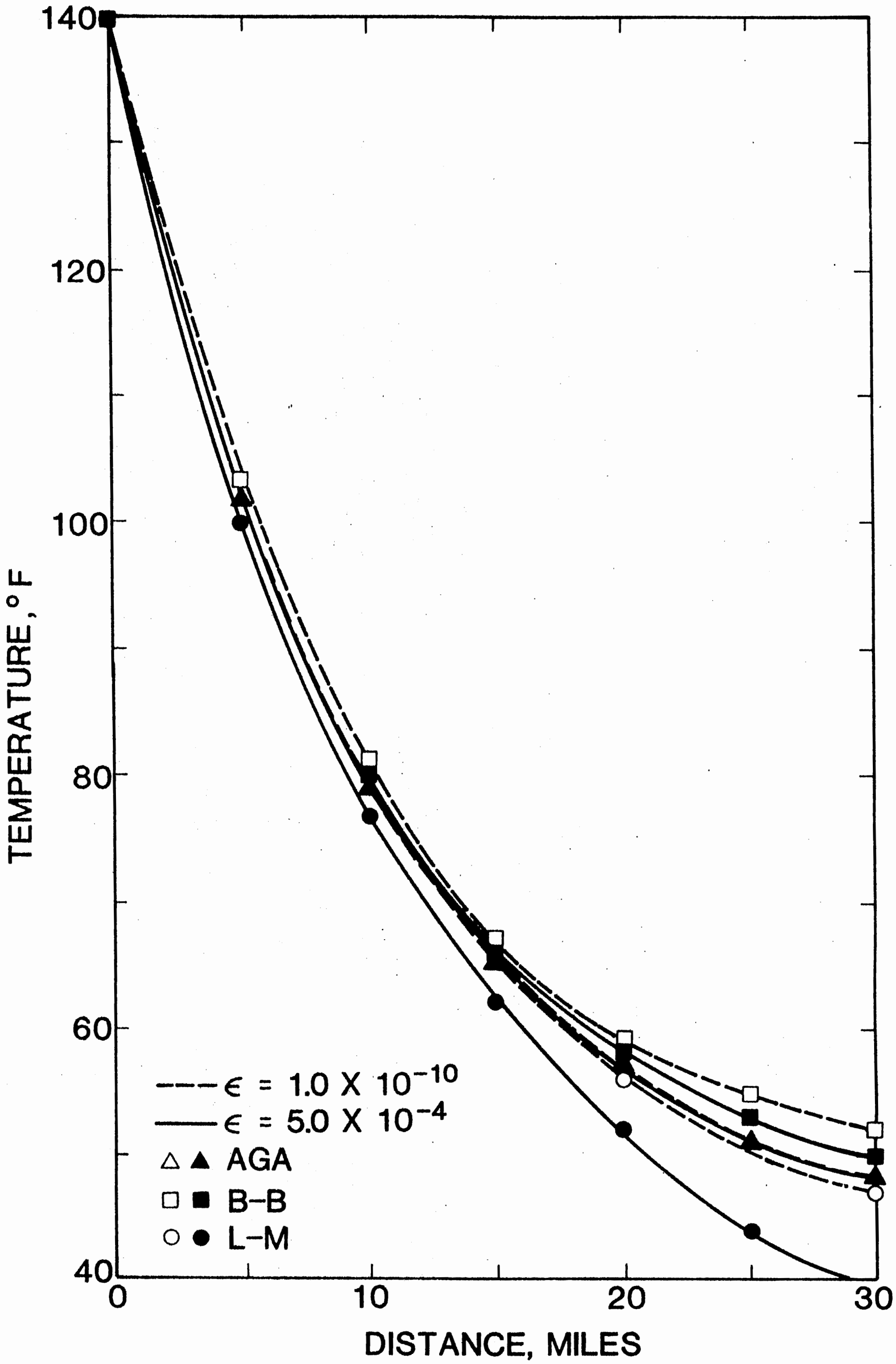

Figure 10. Temperature Profile for Case 1
($U = 1.0$ Btu/hrft^{2}F)

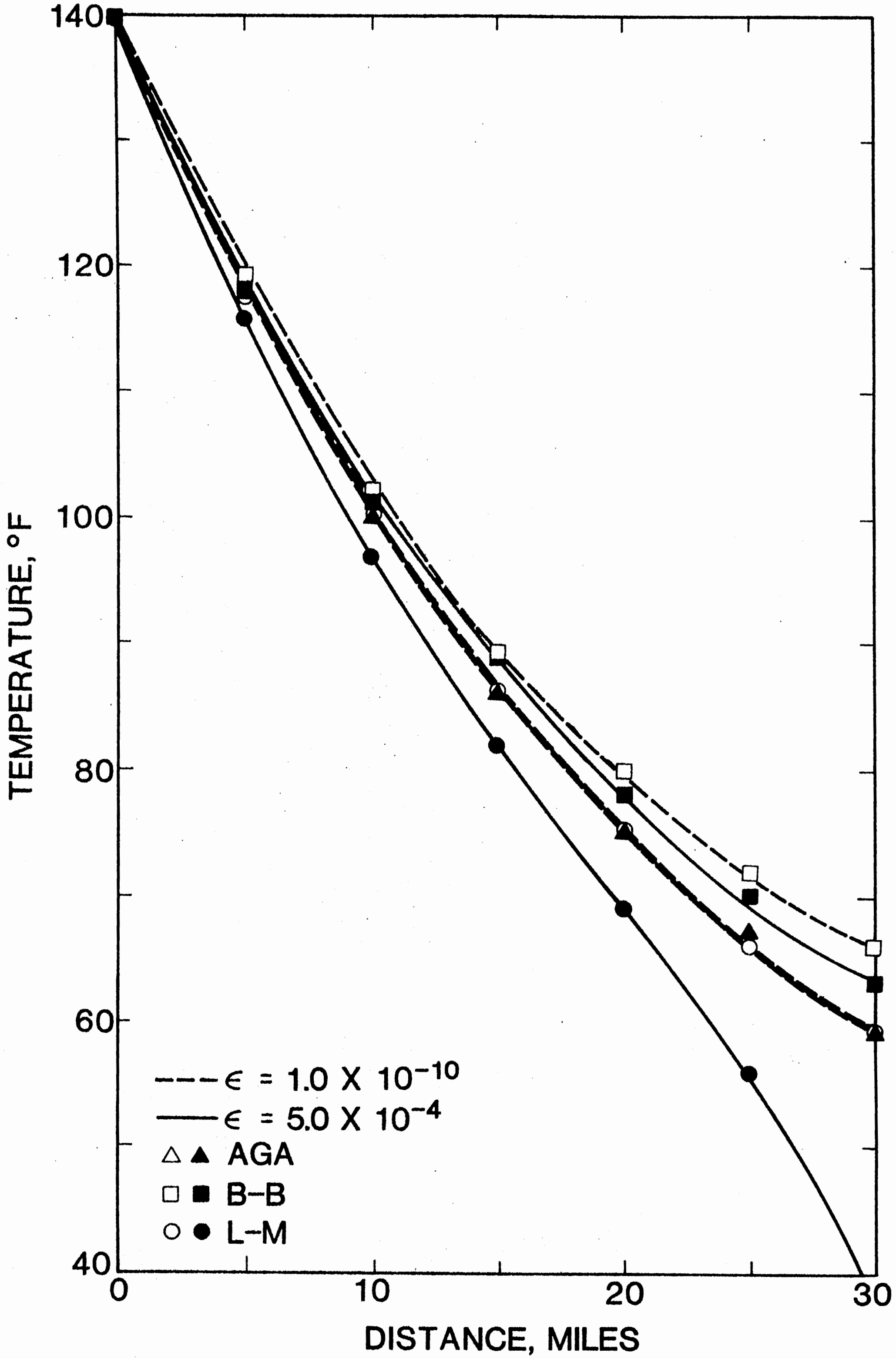

Figure 11. Temperature Profile for Case 1
(U = 0.5 Btu/hrft^{2}F)

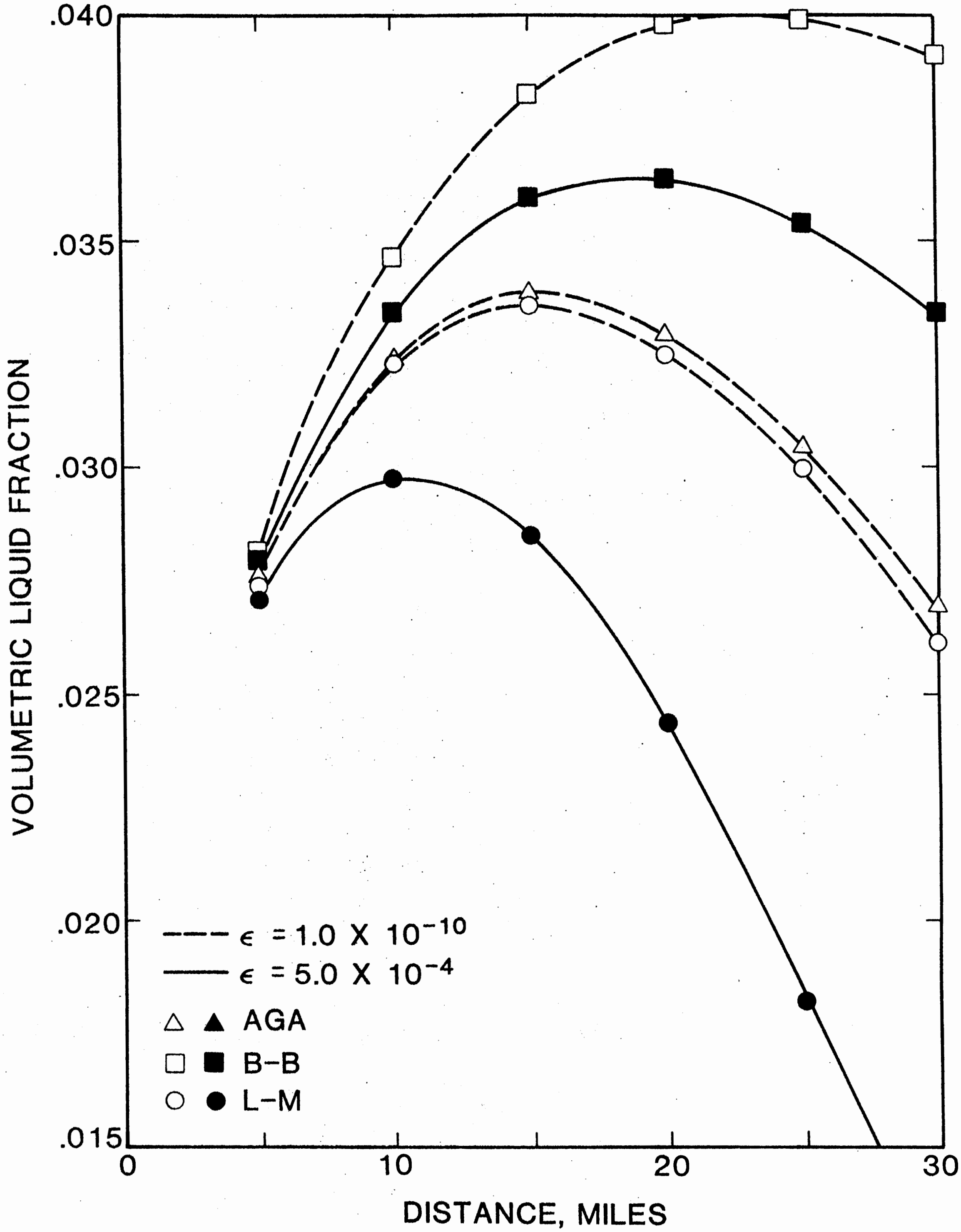

Figure 12. Volumetric Liquid Fraction Profile for Case 1 (U = 1.0 Btu/hrft^{2}F)

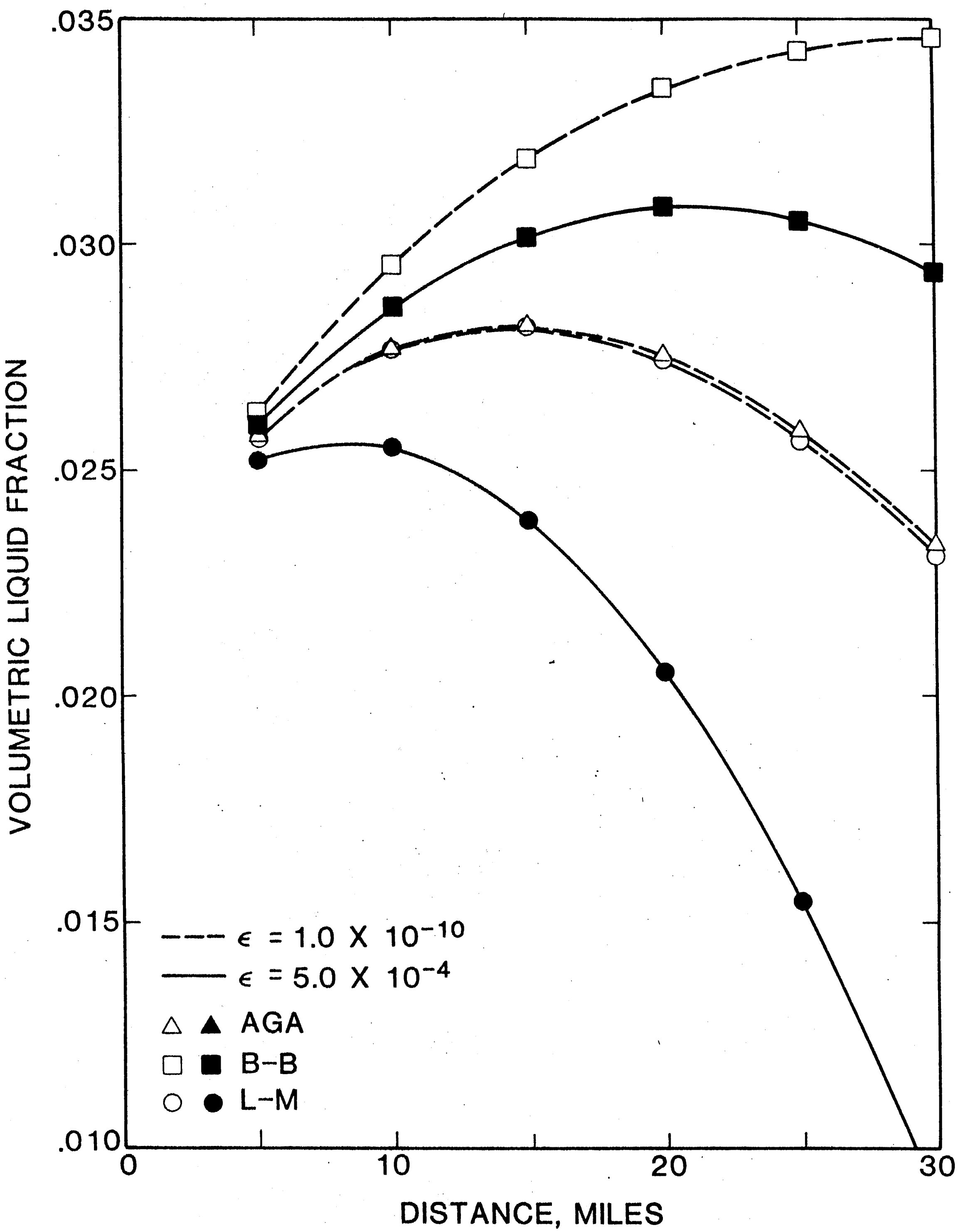

Figure 13. Volumetric Liquid Fraction Profile for Case 1 (U = 0.5 Btu/hrft^{2}F)

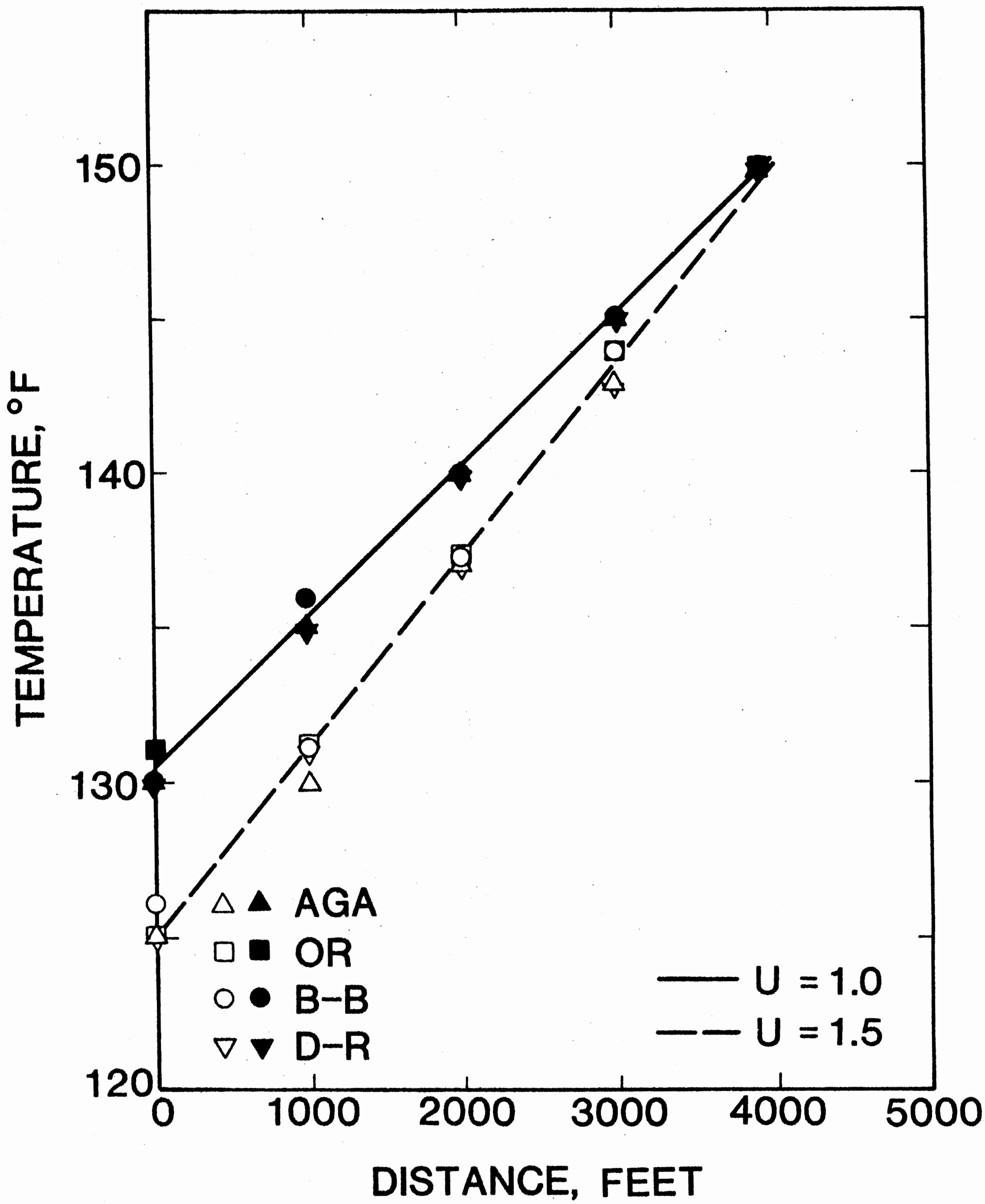

Figure 14. Temperature Profile for Case 2

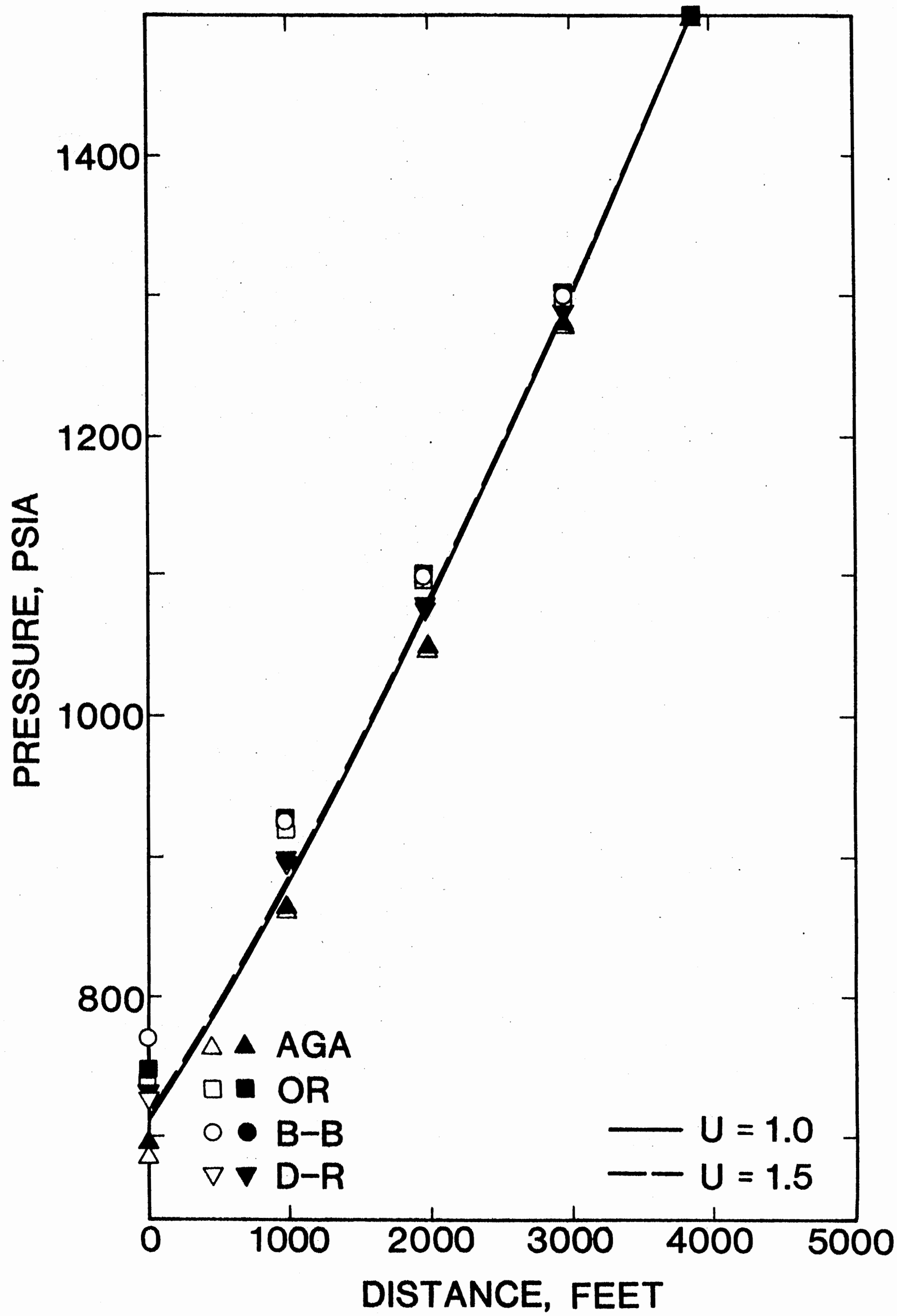

Figure 15. Pressure Profile for Case 2

and Brill was unable to converge in the last segment of the line.

Case 4 was intended as a test case for the horizontal flow methods. A pipeline profile was obtained (32) together with the exact composition of the fluid. Temperature and pressure data were also obtained. Some parameters such as the relative roughness factor and the overall heat transfer coefficient factor were not available. based on available information.

With the temperature profile available, a suitable overall heat transfer coefficient could be chosen that accurately matched the profile. The relative roughness factor was based on the type of pipeline material.

The results, again, were as in Case 1. The AGA and the Lockhart and Martinelli for a smooth pipe predicted essentially the same pressure drop, while Beggs and Brill predicted a lower pressure drop. Lockhart and Martinelli predicted a much higher pressure drop when a rough pipe was used. Figures 19 and 20 clearly illustrate these points.

Case 5 is essentially the same as Case 1, but without elevation change. Since the elevation change in Case 1 (50 ft/mile) is slight, the results of the calculations are almost the same. The absence of liquid heat decreases the pressure drop significantly. In spite of the fact that the angle of inclination in Case 1 is small (0.54^{o}), the pressure loss decreases by about 120 psi for similar cases (Figures 9 and 21).

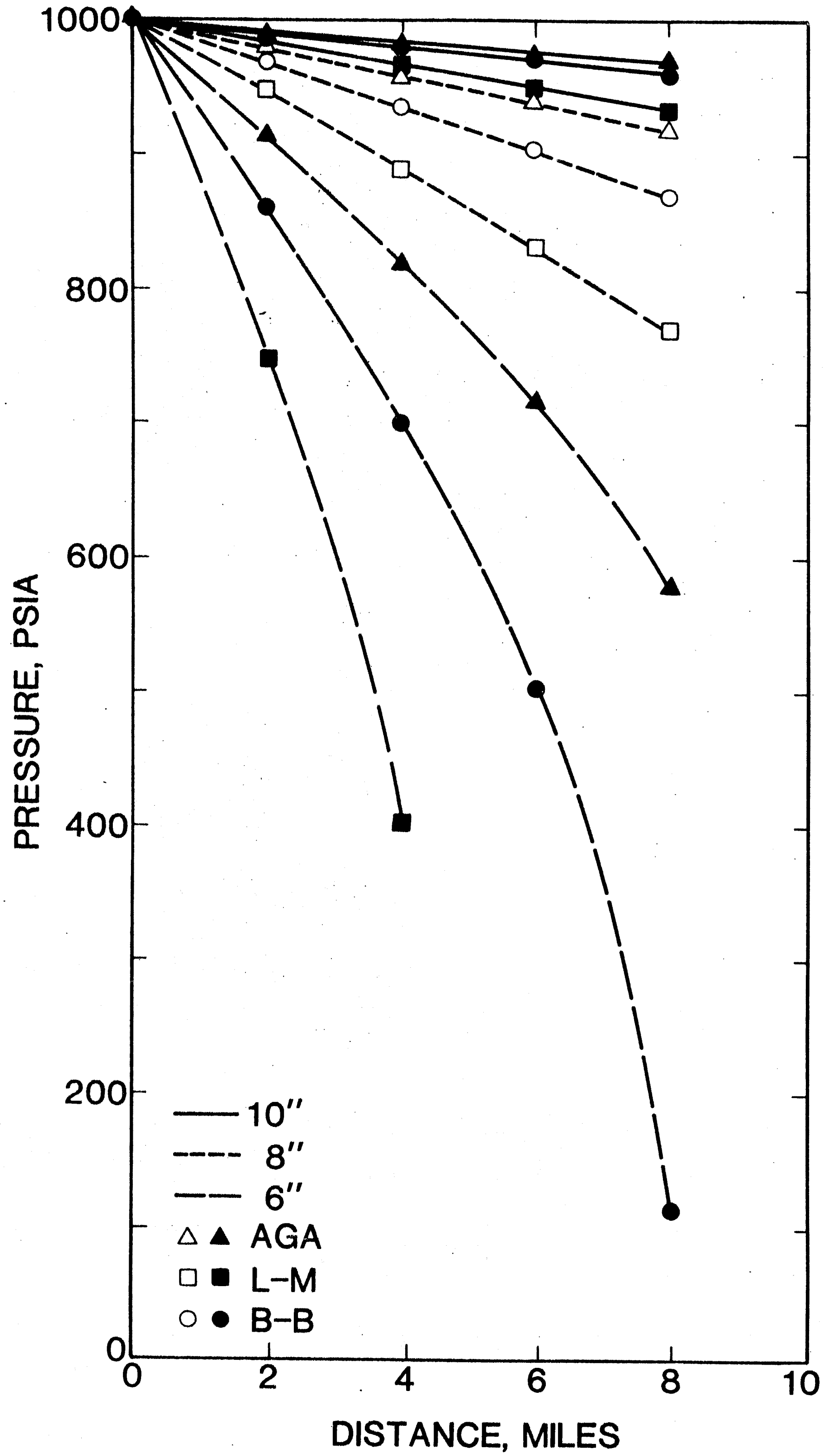

Figure 16. Pressure Profile for Case 3

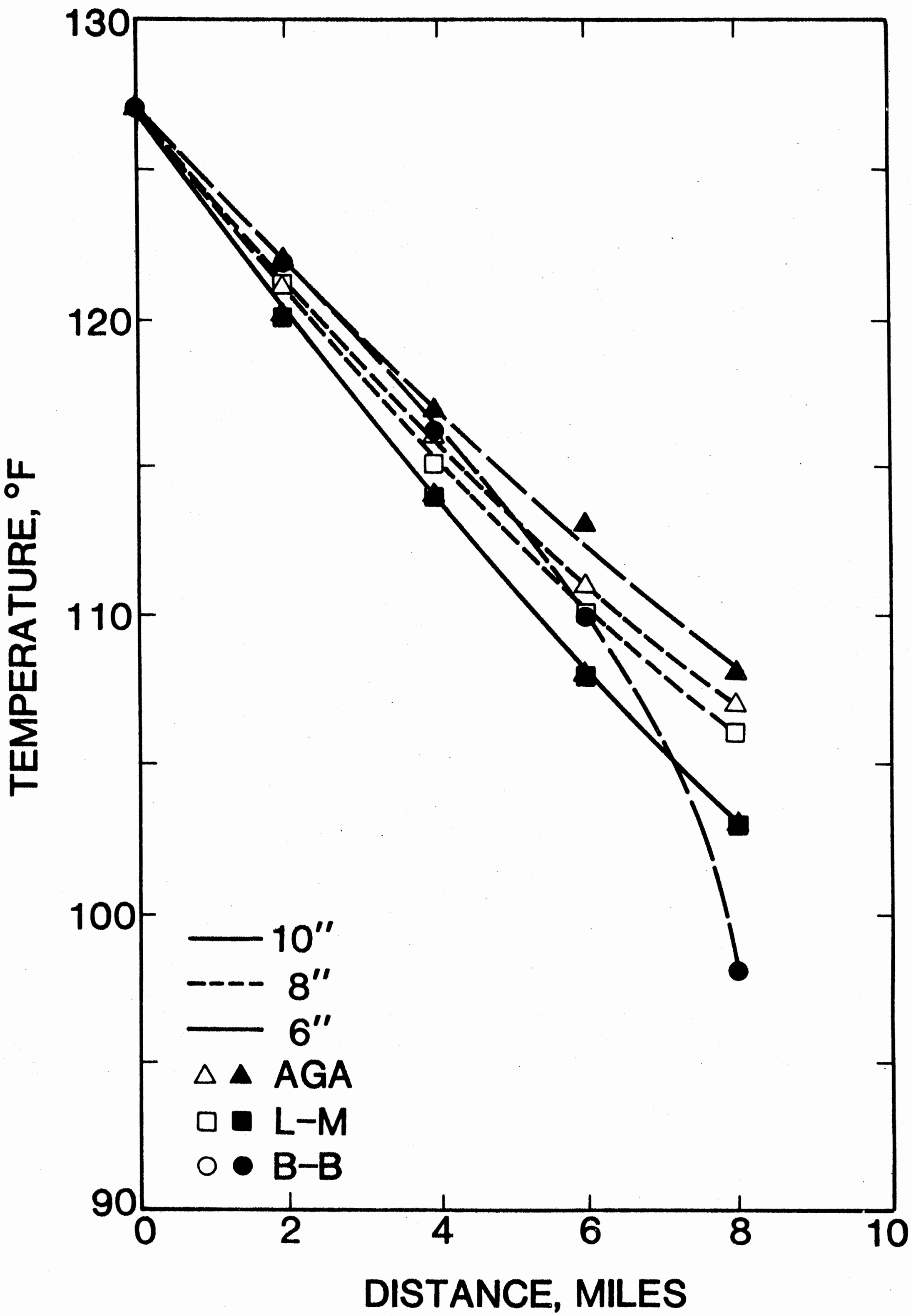

Figure 17. Temperature Profile for Case 3

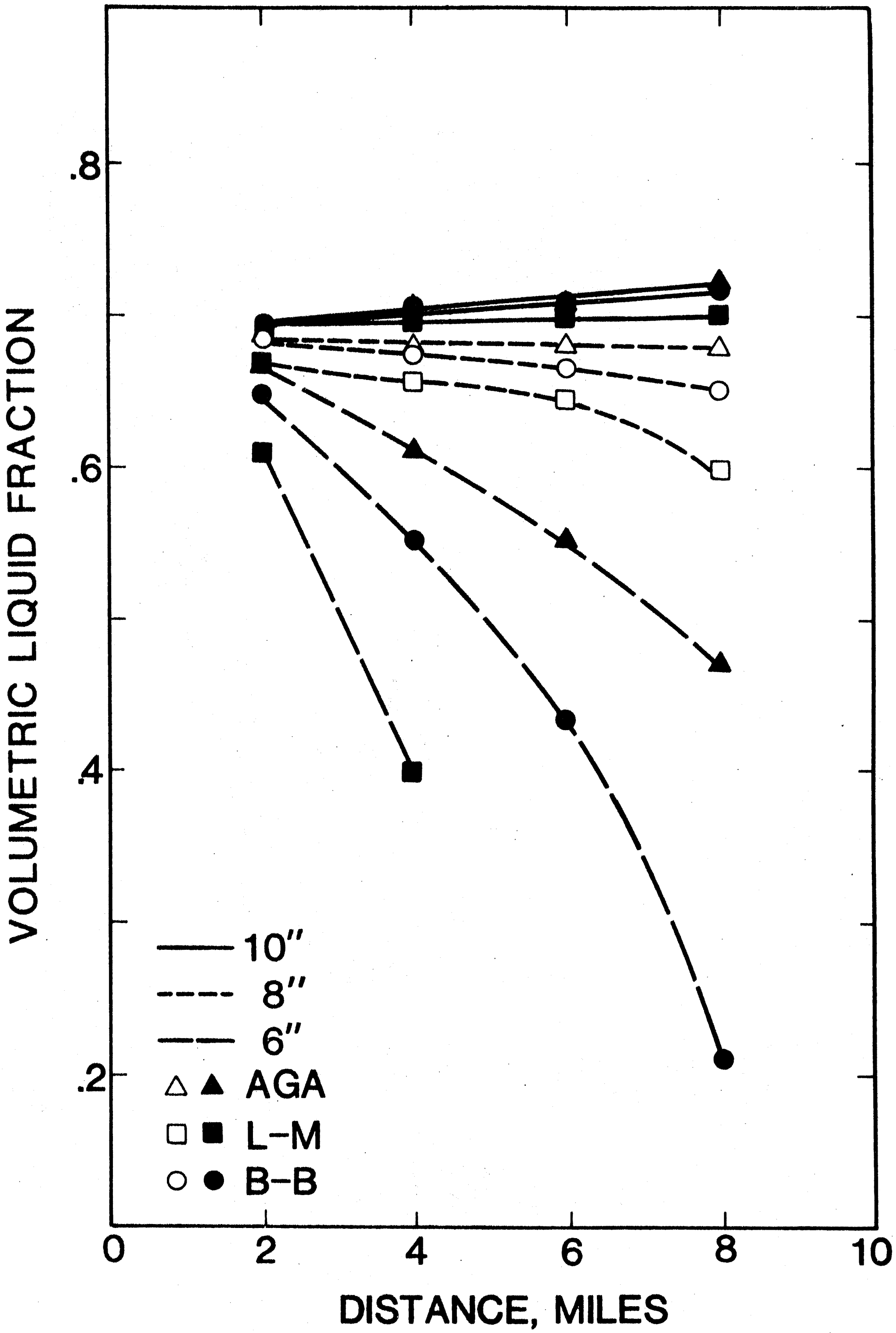

Figure 18. Volumetric Liquid Fraction Profile for Case 3

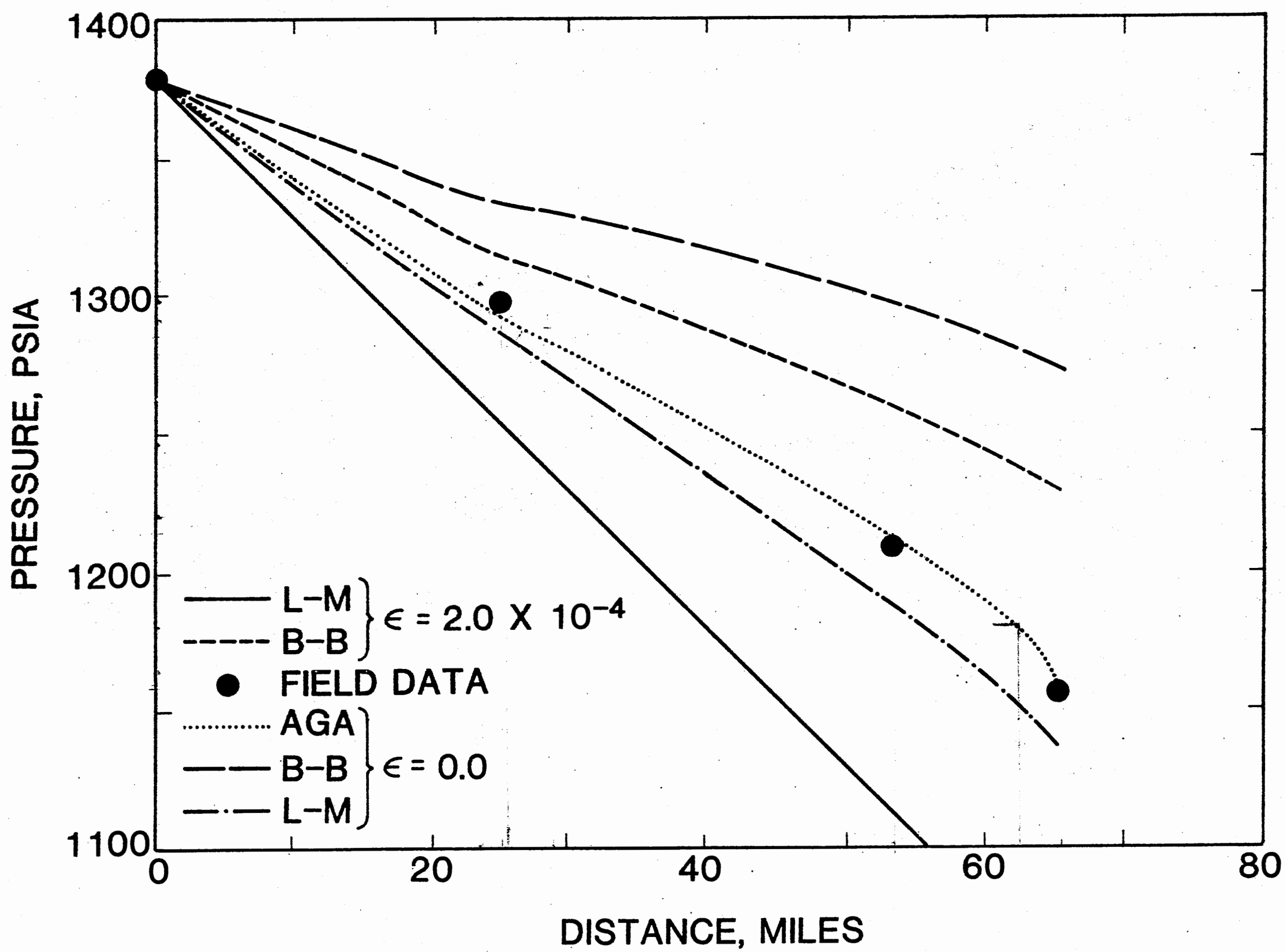

Figure 19. Pressure Profile for Case 4

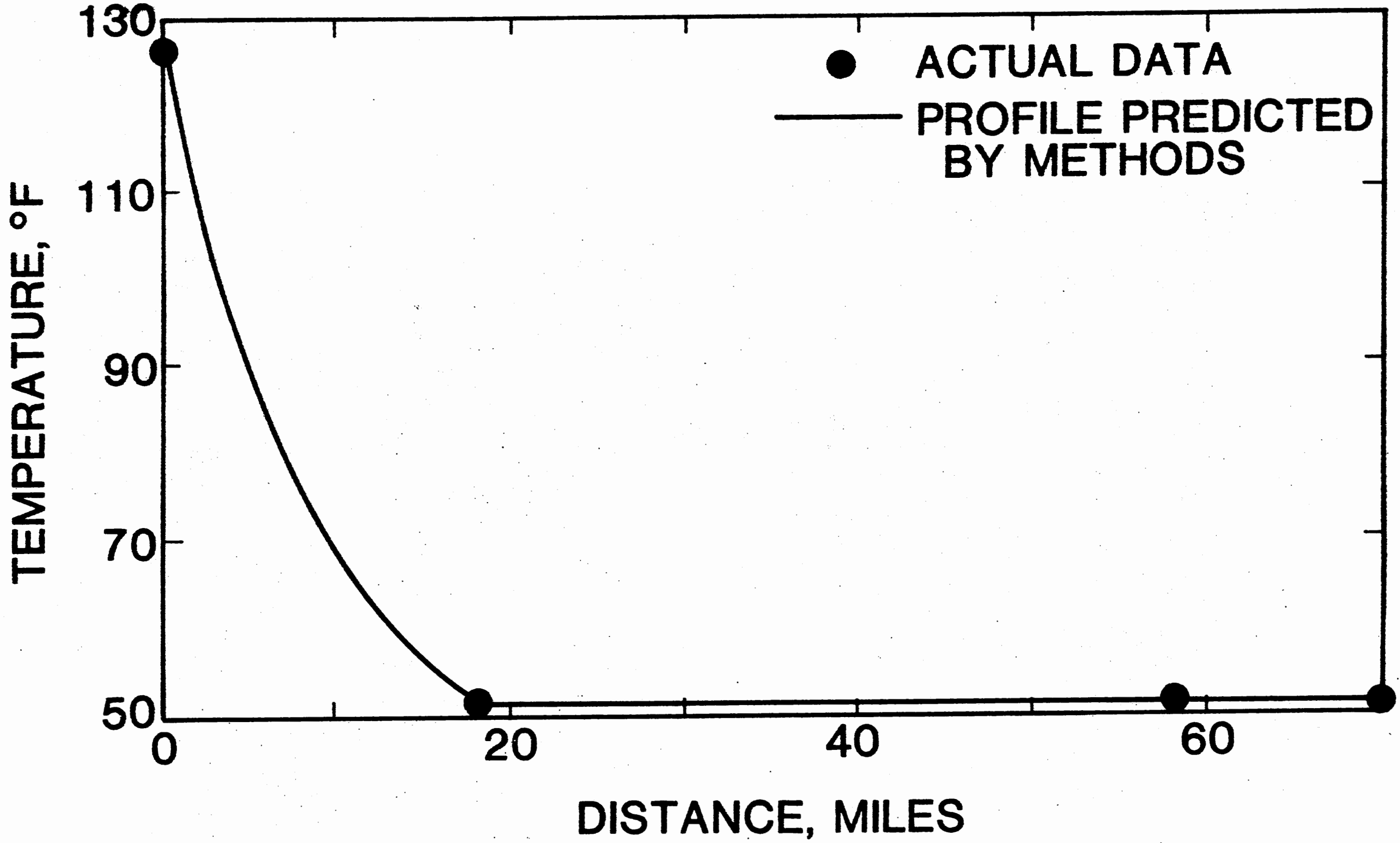

Figure 20. Temperature Profile for Case 4

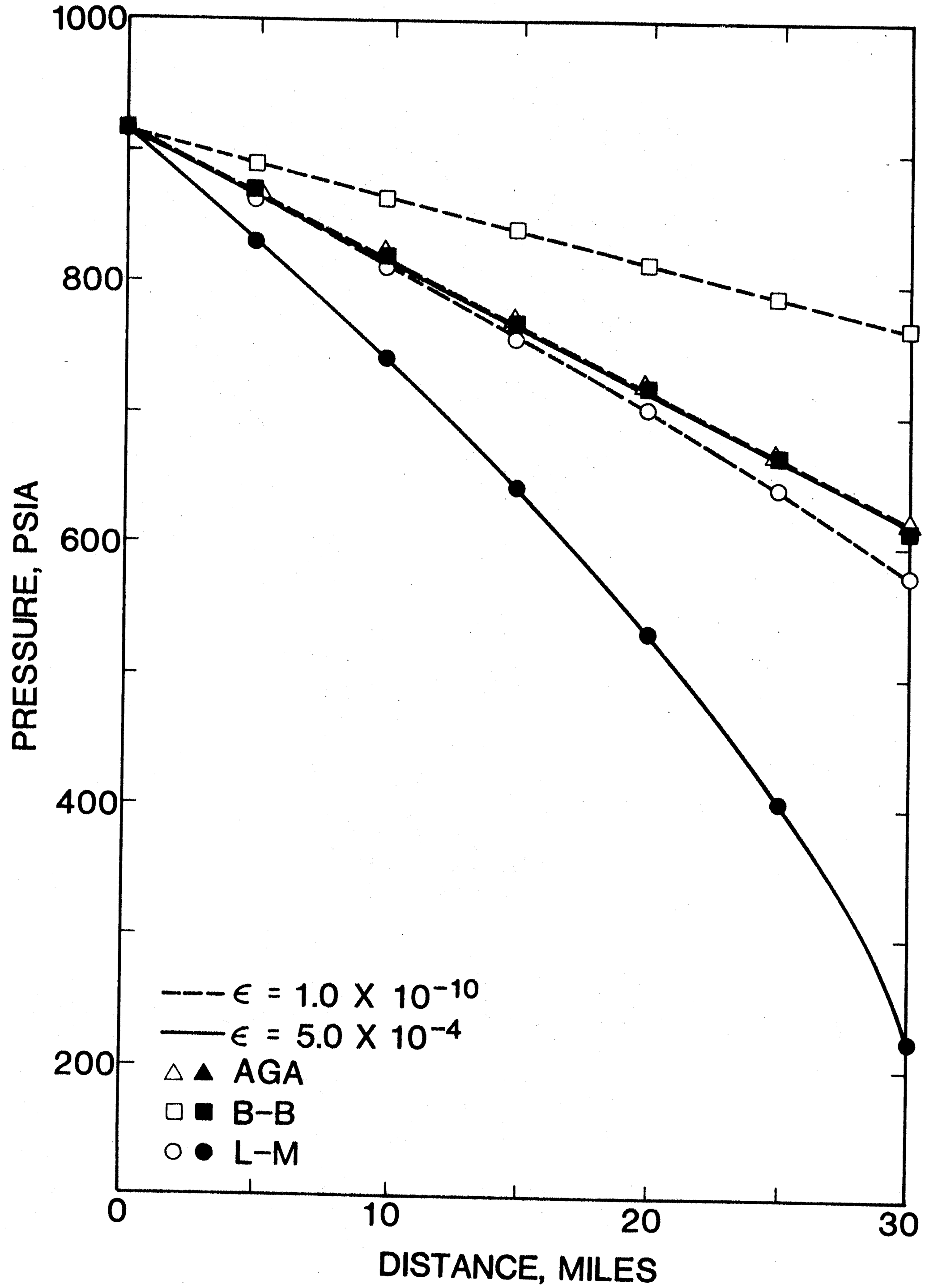

Figure 21. Pressure Profile for Case 5
(U = 0.5 Btu/hrft^{2}F)

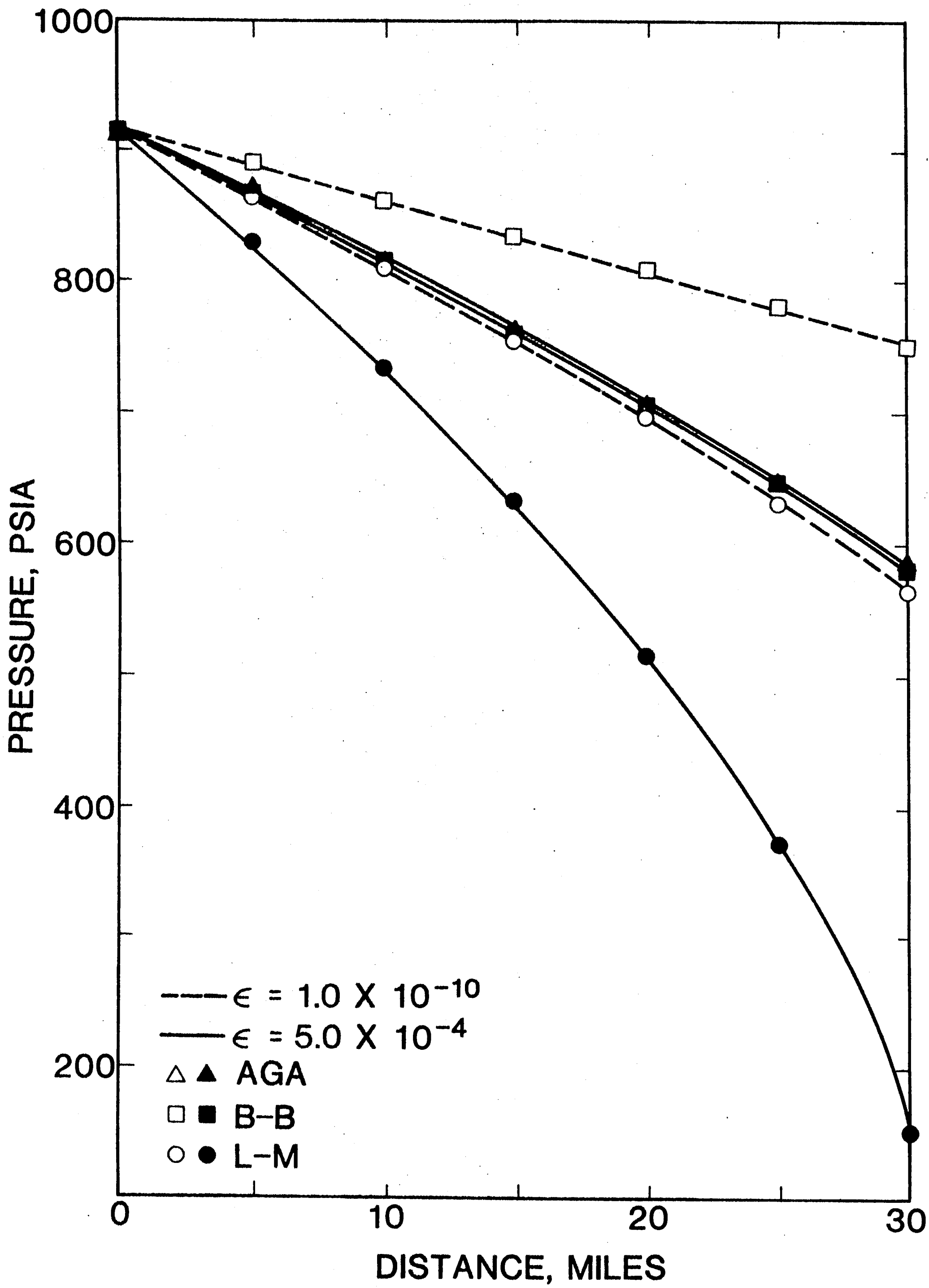

Figure 22. Pressure Profile for Case 5
(U = 0.1 Btu/hrft^{2}F)

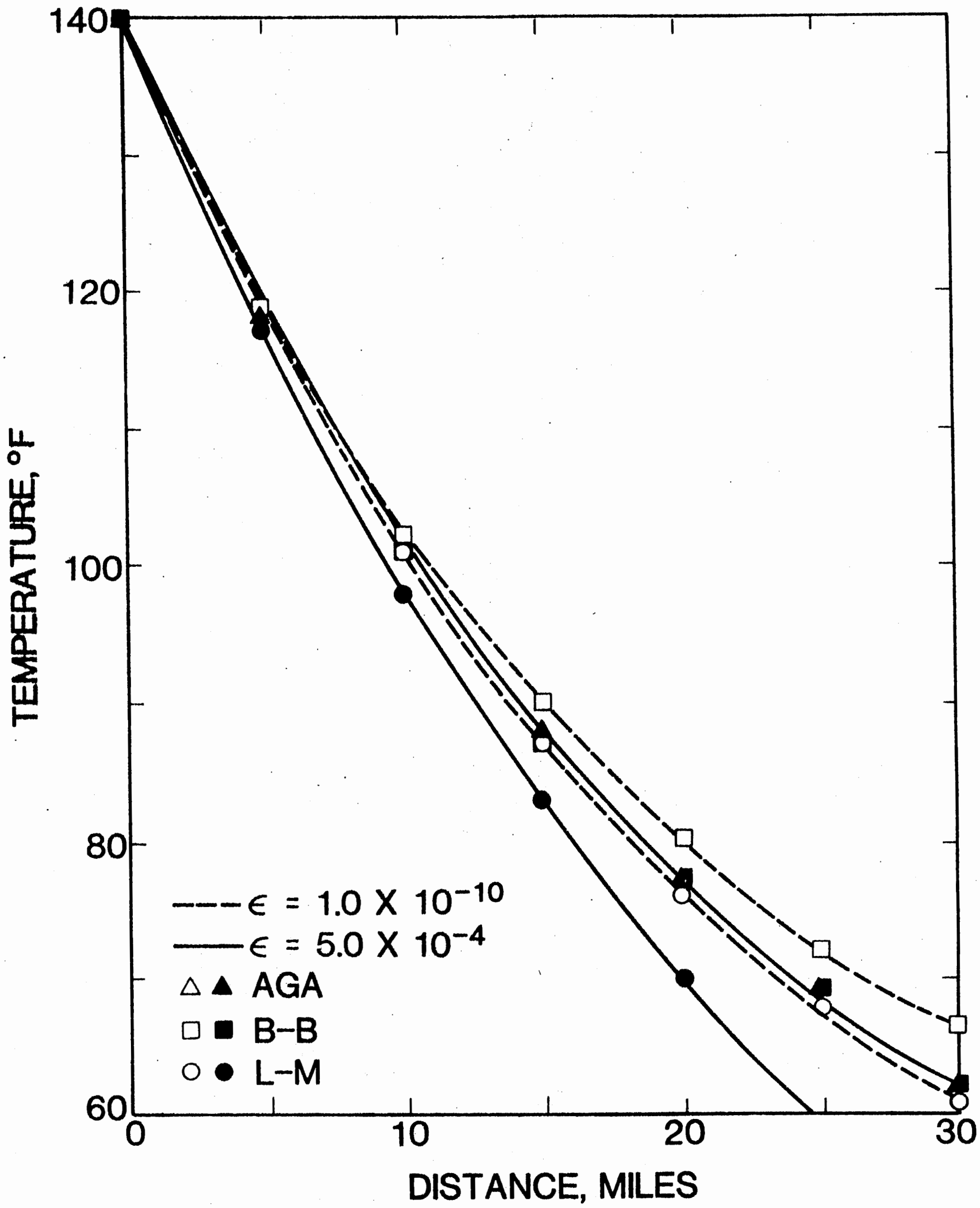

Figure 23. Temperature Profile for Case 5 (U = 0.5 Btu/hrft^{2}F)

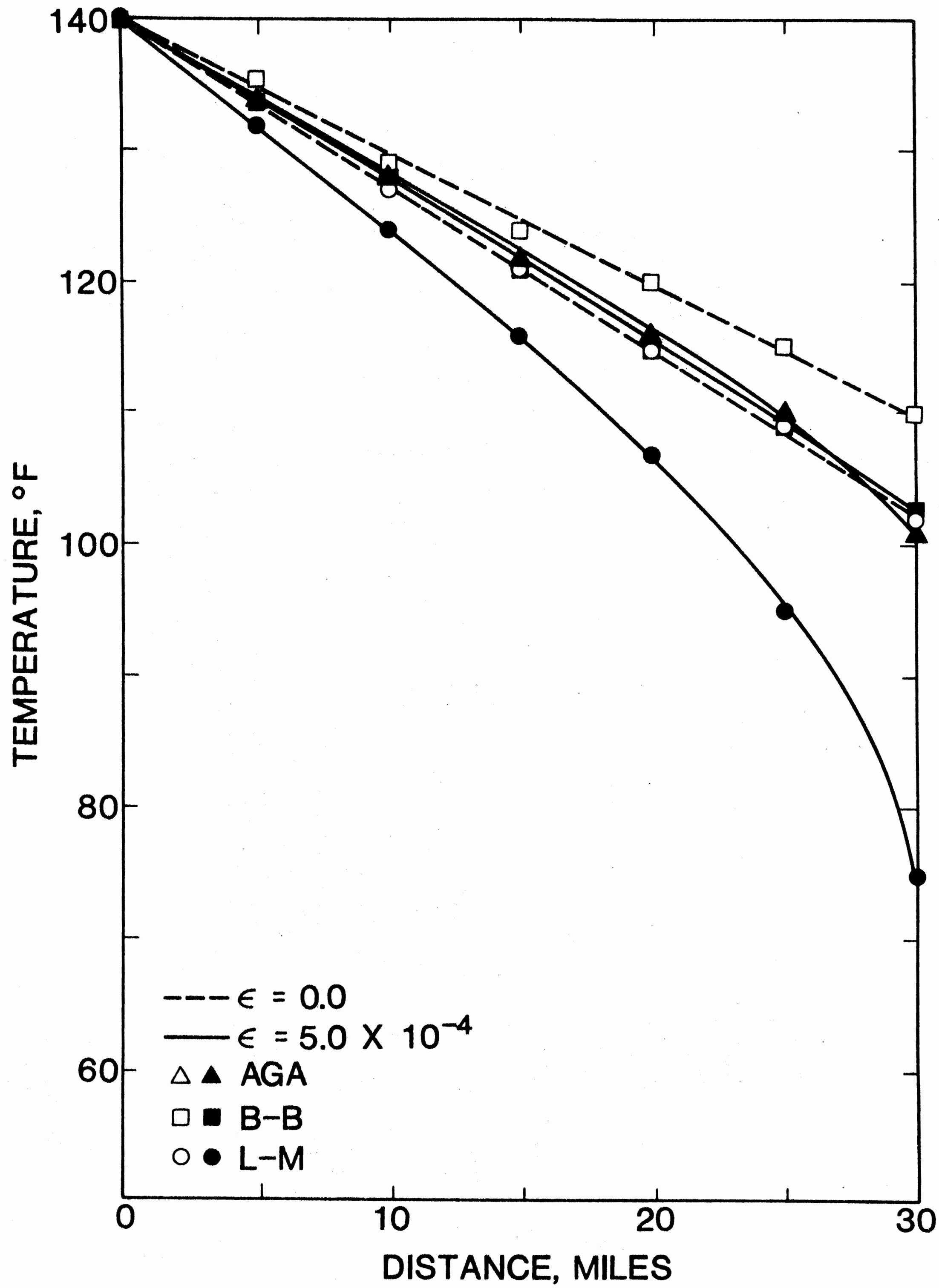

Figure 24. Temperature Profile for Case 5
($U = 0.1$ Btu/hrft^{2}F)

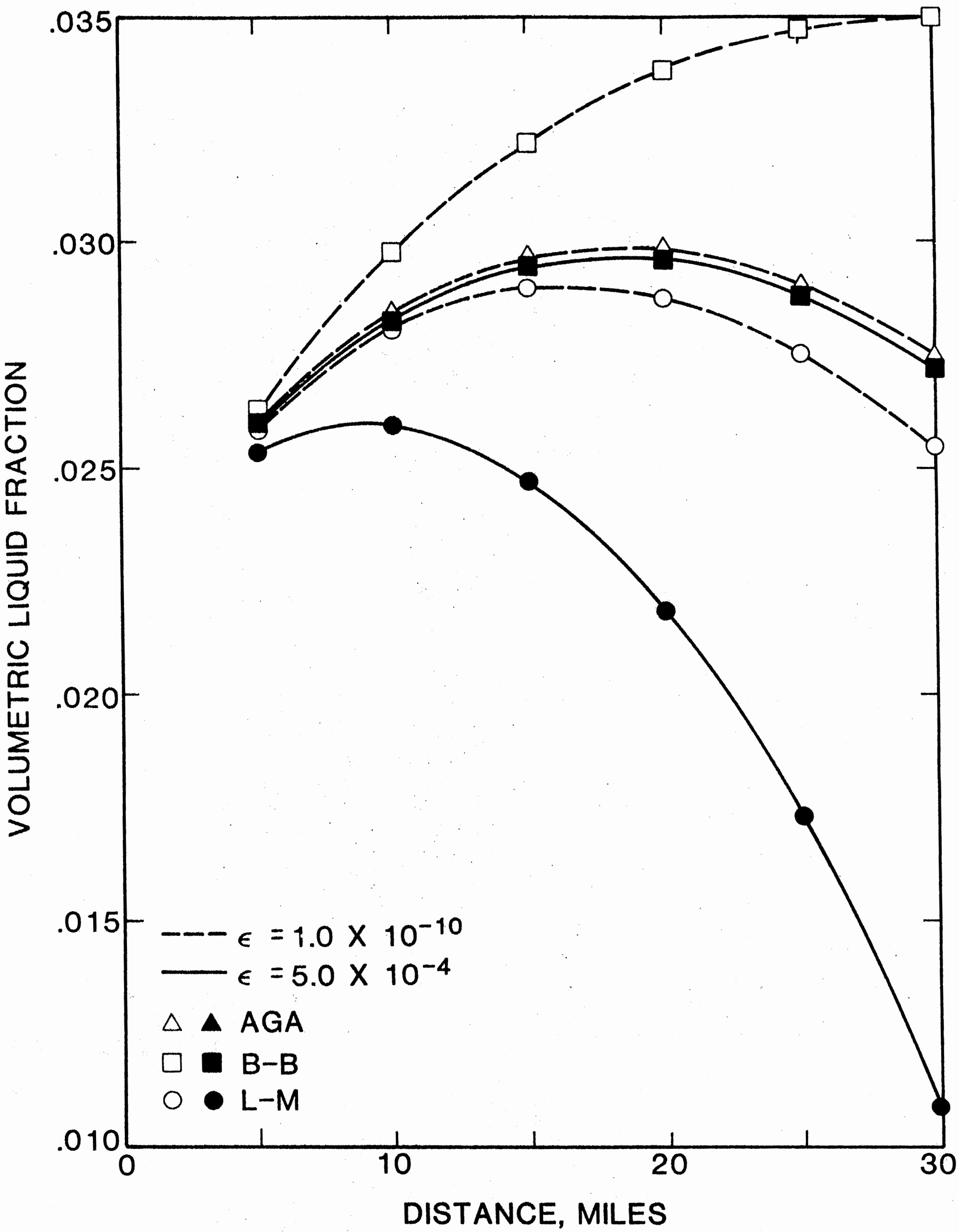

Figure 25. Volumetric Liquid Fraction Profile for Case 5 (U = 0.5 Btu/hrft^{2}F)

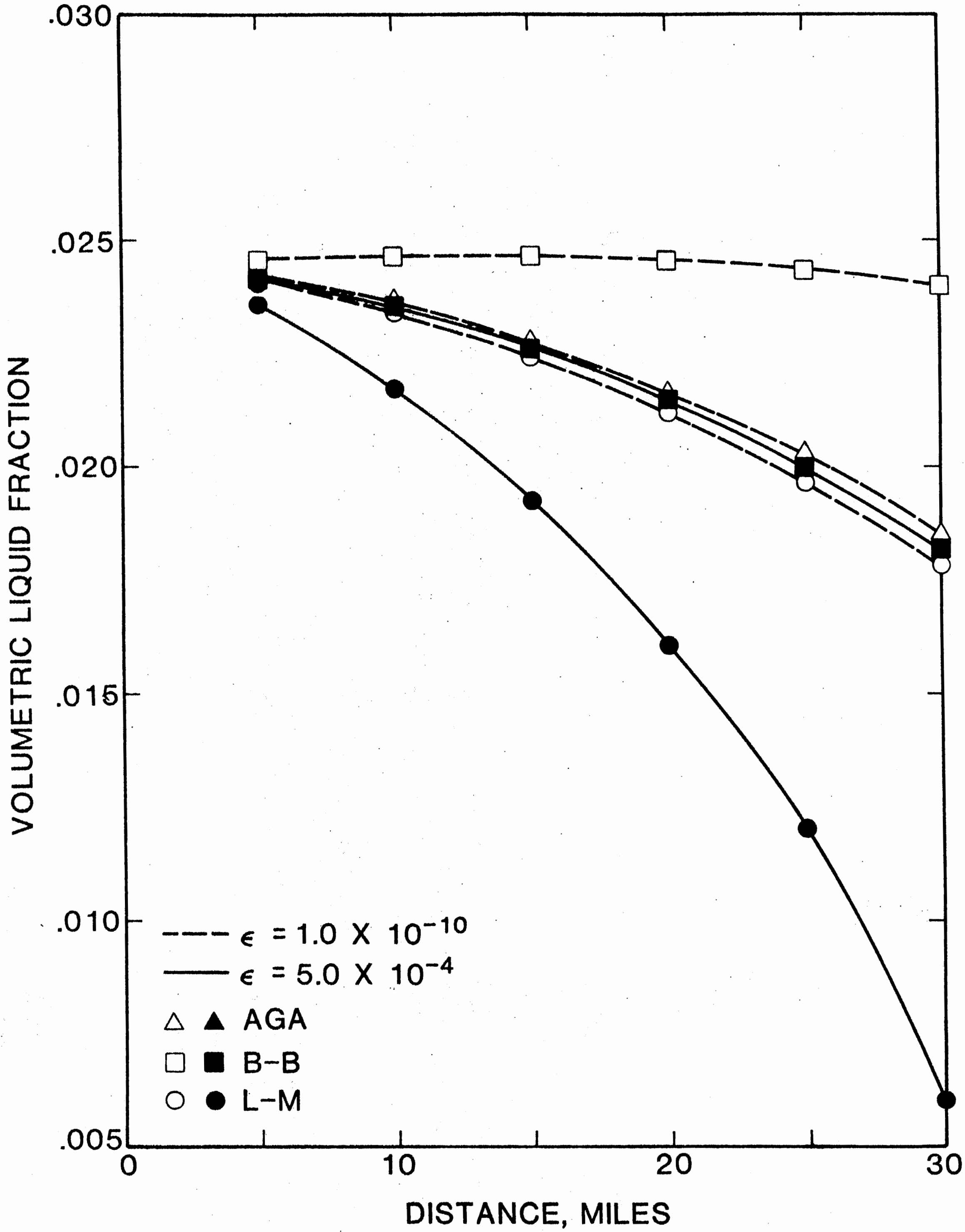

Figure 26. Volumetric Liquid Fraction Profile for Case 5
(U = 0.1 Btu/hrft^{2}F)

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

A computer program was developed that incorporates good physical and thermodynamic properties predictive methods with the two-phase flow calculation methods. Several cases involving a wide range of applications were calculated using the program. The following conclusions were reached as a result of the total study:

1. The computer program is capable of calculating pressure drops in pipelines by different methods with relative ease due to uniform physical and thermodynamic properties.

2. The AGA method seems to be accurate for horizontal flow. Similar results are obtained by Lockhart-Martinelli if smooth pipes are assumed.

4 5

3. The Duns and Ros correlation and the Orkiszewski method predict similar vertical flow pressure drop, accurately matching the results given in Case 2.

4. Empirical correlations are limited by nature to the range of the data. The use of such correlations must be done with the understanding that the results might be in error.

The following recommendations are made for the use of the program and for further studies:

1. The AGA method is recommended for the prediction of pressure drops in horizontal and slightly inclined pipelines.

2. Orkisweski or Duns and Ros are recommended for the prediction of pressure drops in vertical pipelines.

3. An investigation should be conducted using this program to study the effects of viscosity, density, and interfacial tension variations.

4. An experimental investigation of two-phase flow with physical and thermodynamic properties evaluated by an equation of state should be undertaken. Variety of flowing fluids and different flow configurations should be used in order to obtain a reliable and accurate two-phase flow predictive method.

BIBLIOGRAPHY

1. DeGance, A. E. and R. W. Atherton, *Chem. Engr.*, *77*, 135 (1970).
2. DeGance, A. E. and R. W. Atherton, *Chem. Engr.*, *77*, 151 (1970).
3. DeGance, A. E. and R. W. Atherton, *Chem. Engr.*, *77*, 113 (1970).
4. DeGance, A. E. and R. W. Atherton, *Chem. Engr.*, *77*, 95 (1970).
5. DeGance, A. E. and R. W. Atherton, *Chem. Engr.*, *77*, 119 (1970).
6. DeGance, A. E. and R. W. Atherton, *Chem. Engr.*, *77*, 87 (1970).
7. DeGance, A. E. and R. W. Atherton, *Chem. Engr.*, *77*, 101 (1970).
8. DeGance, A. E. and R. W. Atherton, *Chem. Engr.*, *77*, 125 (1971).
9. Lockhart, R. W. and R. C. Martinelli, *Chem. Engr. Prog.*, *45*, 39 (1949).
10. Balasubramanian, G. R., S. B. Koganti and R. Matarajan, *Chem. Engr.*, *87*, 101 (1980).
11. Baker, O., *Oil and Gas J.*, *52*, 185 (1954).
12. Baker, O., *Oil and Gas J.*, 56, 156 (1958).
13. Dukler, A. E., *Gas-Liquid Flow in Pipelines: I. Research Results*, American Gas Association, (1969).
14. Baker, O., et al., *Gas-Liquid Flow in Pipelines: II. Design Manual*, American Gas Association, (1970).
15. Duns, H. and N. C. Ros, *Proc. 6th World Petrol. Cong.*, (1963).
16. Orkiszewski, J., *SPE No. 1546*, (1966).
17. Orkiszewski, J., *J. Petrol. Technol.*, *19*, 829 (1967).
18. Beggs, D. H. and J. P. Brill, *SPE No. 4007*, (1972).
19. Beggs, D. H. and J. P. Brill, *J. Petrol. Technol.*, *25*, 606 (1973).
20. Payne, G. A., et al., *J. Petrol. Technol.*, *31*, 1198 (1979).

21. Danesh, A., J. Petrol. Technol., 32, 169 (1980).

22. Moshfeghian, M., Shiraz University, Private Communication (1980).

23. Wallis, G. B., One Dimensional Two-Phase Flow, McGraw-Hill Book Co., (1969).

24. Hewitt, G. F., Measurement of Two-Phase Flow Parameters, Academic Press, (1978).

25. Campbell, J. M., Gas Conditioning and Processing, Vol. 1, Campbell Petroleum Series, (1978).

26. Campbell, J. M., Gas Conditioning and Processing, Vol. 2, Campbell Petroleum Series, (1978).

27. Erbar, J. H. and R. N. Maddox, Gas Conditioning and Processing, Vol. 3, Campbell Petroleum Series, (1978).

28. Erbar, J. H. and R. N. Maddox, Oklahoma State University, Private Communication (1980).

29. Ried, R. C., et al., The Properties of Gases and Liquids, McGraw-Hill Book Co., (1977).

30. Gas Processors Supplier Association, Engineering Data Book, (1979).

31. Gould, T. L., J. Petrol. Technol., 31, 313 (1979).

32. Erbar, J. H., Oklahoma State University, Private Communication (1980).

33. Gangriwal, H. A., Pressure Drop in Two-Phase Flow, (1978).

34. Brookbank, M. R. and P. M. J. Fagiano, SPE No. 5264, (1975).

35. Diehl, J. E., Petrol. Refiner, 36, 147 (1957).

36. Hoodendroon, C. J. and A. A. Buitelaar, Chem. Engr. Sci., 16, 208 (1961).

APPENDIX A

RESULTS OF CALCULATIONS FOR CASE 1

TABLE VIII

VOLUMETRIC LIQUID FRACTION FOR CASE 1

Segment No.	AGA	Lockhart-Martinelli $\varepsilon/D = 0.0$	Lockhart-Martinelli $\varepsilon/D = 4.0 \times 10^{-4}$	Beggs and Brill $\varepsilon/D = 0.0$	Beggs and Brill $\varepsilon/D = 4.0 \times 10^{-4}$
			($U = 0.5$ Btu/hrft^{2}F)		
1	.02578	.02578	.02521	.026304	.026039
2	.02767	.02765	.02553	.029586	.028665
3	.02821	.02813	.02391	.031933	.030234
4	.02757	.02742	.02054	.033466	.030827
5	.02591	.02568	.01549	.034321	.03057
6	.02337	.02308	.06808	.034599	.029464
			($U = 1.0$ Btu/hrft^{2}F)		
1	.02764	.02741	.02701	.028194	.027901
2	.03234	.03231	.02979	.03462	.033439
3	.03386	.03361	.02852	.638251	.035966
4	.03294	.03249	.02439	.039772	.036397
5	.03045	.02979	.01825	.039886	.035434
6	.02695	.02613	.01011	.039108	.033469

TABLE IX

PRESSURE AND TEMPERATURE PROFILES FOR CASE 1
(U = 1.0 Btu/hrft^{2}F)

Segment No.	AGA	Lockhart-Martinelli ε = 0.0	Lockhart-Martinelli ε = $5x10^{-4}$	Beggs and Brill ε = 0.0	Beggs and Brill ε = $5x10^{-4}$
			Pressure Psia		
1	857	857	823	887	871
2	800	797	727	860	828
3	742	736	625	832	786
4	682	673	510	804	747
5	618	606	373	777	705
6	549	535	175	751	661
			Temperature ^oF		
1	102	102	100	103	102
2	79	79	77	81	80
3	65	65	62	67	66
4	57	56	52	59	58
5	51	51	44	55	53
6	48	47	32	52	50

TABLE X

PRESSURE AND TEMPERATURE PROFILES FOR CASE 1
($U = 0.5$ Btu/hrft^{2}F)

Segment No.	AGA	Lockhart-Martinelli $\varepsilon = 0.0$	Lockhart-Martinelli $\varepsilon = 5x10^{-4}$	Beggs and Brill $\varepsilon = 0.0$	Beggs and Brill $\varepsilon = 5x10^{-4}$
			Pressure Psia		
1	857	857	823	888	872
2	797	797	725	861	831
3	737	734	618	834	791
4	673	669	499	808	749
5	606	601	354	782	705
6	534	528	132	757	657
			Temperature ^oF		
1	118	118	116	119	118
2	100	100	97	102	101
3	86	86	82	89	88
4	75	75	69	80	78
5	67	66	56	72	70
6	59	59	36	66	63

APPENDIX B

RESULTS OF CALCULATIONS FOR CASE 2

TABLE XI

PRESSURE AND TEMPERATURE PROFILES FOR CASE 2

	Duns and Ross		Beggs and Brill		Orkiszewski		AGA	
	U = 1.0	U = 1.5	U = 1.0	U = 1.5	U = 1.0	U = 1.5	U = 1.0	U = 1.5
ΔP								
1	1290	1289	1297	1297	1302	1301	1280	1280
2	1083	1081	1100	1098	1104	1101	1061	1059
3	899	896	926	0923	923	919	864	861
4	729	724	774	769	749	742	691	686
ΔT								
1	145	143	145	144	145	144	145	143
2	140	137	140	137	135	131	135	130
3	135	131	136	131	135	131	135	130
4	130	125	131	126	131	125	130	125

APPENDIX C

RESULTS OF CALCULATIONS FOR CASE 3

TABLE XII

RESULTS OF CASE 3

Method	American Gas Assoc.-API			Lockhart and Martinelli			Beggs and Brill		
	10"	8"	6"	10"	8"	6"	10"	8"	6"
ΔP Psia									
1	993	980	914	982	946	748.7	990	968	860
2	986	959	820	965	890	391.5	979	936	700
3	980	939	712	948	832	--*	969	903	501
4	973	918	581	931	771	--*	959	870	111 *
ΔT ^{o}F									
1	120	121	122	120	121	120	120	121	122.0
2	114	116	117	114	115	118	114	116	116.0
3	108	111	113	108	110	--*	108	111	110.0
4	103	107	108	103	106	--*	103	107	98. *
Vol. Liq. Fr.									
1	.69459	.68850	.66610	.69098	.67780	.61168	.69332	.68482	.64837
2	.70565	.6879	.61547	.69533	.65406	.40252	.70217	.67646	.55502
3	.71577	.68610	.55266	.69827	.62609	--*	.70983	.66628	.43048
4	.72468	.68304	.47125	.69975	.59332	--*	.71618	.6542	.20950 *

*No Convergence for this segment

APPENDIX D

RESULTS OF CALCULATIONS FOR CASE 4

TABLE XIII

PRESSURE AND TEMPERATURE PROFILES FOR CASE 4

Segment Leng.	Segment No.	AGA		Beggs and Brill ε = 0.0	Beggs and Brill ε = .0002	Lockhart and Martinelli ε = 0.0	Lockhart and Martinelli ε = .0002	Elevation	Length Miles
				Pressure Psia					
6.0	1	1356	1356	1368	1363	1357	1349	36	6.0
6.0	2	1336	1336	1357	1348	1335	1319	36	6.0
6.0	3	1315	1315	1346	1333	1312	1282	36	6.0
5.55	4	1296	1301	1336	1319	1290	1259	39	5.55
5.0	1	1283	1269	1330	1310	1274	1236	0	5.0
4.48	2	1270	1275	1324	1301	1258	1215	0	4.48
5.0	1	1256	1260	1317	1291	1241	1191	0	5.0
5.0	2	1242	1244	1311	1281	1224	1167	0	5.0
5.0	3	1228	1228	1304	1271	1206	1142	0	5.0
5.35	4	1212	1211	1297	1260	1187	1114	0	5.35
5.0	5	1192	1191	1286	1247	1166	1085	64	5.0
5.0	6	1173	1175	1276	1233	1145	1056	64	5.0
1.46	7	1160		1273	1228	1138	1046	19	1.46

TABLE XIII (Continued)

Segment Leng.	Segment No.	AGA	Beggs and Brill $\varepsilon = 0.0$	Beggs and Brill $\varepsilon = .0002$	Lockhart and Martinelli $\varepsilon = 0$	Lockhart and Martinelli $\varepsilon = .0002$	Elevation	Length Miles
			Temperature $^{\circ}F$					
	1	83 87.7	84	84	83	83	Same as for Pressure	Same as for Pressure
	2	65 68.2	65	65	65	64		
	3	56 58.6	57	56	56	56		
	4	53 54.6	53	53	52	52		
	1	52 52.4	52	52	51	51		
	2	51 51.2	52	51	50	50		
	1	51 50.4	52	52	50	50		
	2	51 49.8	53	52	50	50		
	3	51 49.2	53	53	51	50		
	4	52 48.8	53	53	51	50		
	5	52 48.8	53	53	51	50		
	6	52 48.1	53	53	51	50		
	7	52	53	53	51	50		

APPENDIX E

RESULTS OF CALCULATIONS FOR CASE 5

TABLE XIV

VOLUMETRIC LIQUID FRACTION FOR CASE 5

Segment No.	AGA	Lockhart-Martinelli $\varepsilon/D = 0.0$	Lockhart-Martinelli $\varepsilon/D = 4.0x10^{-4}$	Beggs and Brill $\varepsilon/D = 0.0$	Beggs and Brill $\varepsilon/D = 4.0x10^{-4}$
			(U = 0.1 $Btu/hrft^2F$)		
1	.02422	.02416	.02364	.024544	.024203
2	.02362	.02344	.02171	.02468	.023571
3	.02277	.02245	.01923	.024692	.02268
4	.02165	.02120	.0161	.024582	.021517
5	.02026	.01966	.01214	.024351	.02006
6	.01857	.01784	.0061	.02400	.018279
			(U = 0.5 $Btu/hrft^2F$)		
1	0.02598	0.02591	0.02534	0.026335	0.025968
2	0.02841	0.02811	0.02598	0.029704	0.028359
3	0.02965	0.02903	0.02477	0.032172	0.029554
4	0.02983	0.02878	0.02187	0.033836	0.029687
5	0.02908	0.02756	0.01735	0.034787	0.028881
6	0.0275	0.02549	0.01092	0.035122	0.027247

TABLE XV

PRESSURE AND TEMPERATURE PROFILES FOR CASE 5
(U = 0.5 Btu/hrft^{2}F)

Segment No.	AGA	Lockhart-Martinelli $\varepsilon = 0.0$	Lockhart-Martinelli $\varepsilon = 5x10^{-4}$	Beggs and Brill $\varepsilon = 0.0$	Beggs and Brill $\varepsilon = 5x10^{-4}$
			Pressure Psia		
1	869	864	830	868	890
2	822	812	740	720	865
3	775	757	642	772	840
4	725	701	532	722	815
5	674	641	401	670	791
6	619	577	222	614	765
			Temperature ^oF		
1	118	118	117	118	119
2	101	101	98	101	102
3	88	87	83	87	90
4	77	76	70	72	80
5	69	68	58	69	72
6	62	61	44	62	66

TABLE XVI

PRESSURE AND TEMPERATURE PROFILES FOR CASE 5
($U = 0.1$ Btu/hrft^{2}F)

Segment No.	AGA	Lockhart-Martinelli $\varepsilon = 0.0$	Lockhart-Martinelli $\varepsilon = 5x10^{-4}$	Beggs and Brill $\varepsilon = 0.0$	Beggs and Brill $\varepsilon = 5x10^{-4}$
			Pressure Psia		
1	868	864	830	889	867
2	818	811	737	863	816
3	766	755	634	836	763
4	711	695	516	809	706
5	651	632	372	781	644
6	587	564	151	752	577
			Temperature ^oF		
1	134	134	132	135	134
2	128	127	124	124	128
3	122	121	116	124	121
4	116	115	107	120	115
5	110	109	95	115	109
6	103	102	75	110	103

VITA

Saedeldeen Ahmad Akashah

Candidate for the Degree of

Doctor of Philosophy

Thesis: PRESSURE DROP IN TWO-PHASE FLOW

Major Field: Chemical Engineering

Biographical:

Personal Data: Born in Nablus, Palestine, June 26, 1947, the son of Ahmad M. Akashah and Maneafa Hafiz Sharabi-Akashah.

Education: Graduated from Shawaikh High School, Kuwait, in 1965; graduated from Cate Preperatory School, Carpentaria, California, 1966; attended University of California at Santa Barbara, 1966-1970; obtained Bachelor of Science in Chemical Engineering degree from Tri-State University, Angola, Indiana, August, 1972; obtained Master of Science degree from Oklahoma State University, May, 1978; completed the requirements for Doctor of Philosophy degree in Chemical Engineering in December, 1980.

Professional Experience: Employed as Science Tutor in Refugee Camps in Lebanon, April-September, 1970; employed as Industrial Planner by the Industrial Development and Consulting Bureau, Ministry of Commerce and Industry, Kuwait, 1972-1975; employed by the University of Kuwait 1975-present.

Professional Membership and Honors: Member, Society of Petroleum Engineers; Associate Member American Institute of Chemical Engineers; Member Kuwait Graduate Society; Member Kuwait Society of Engineers; United Nations Fellow, 1973; United Nations Fellow, 1975; Member Omega Chi Epsilon; Member Tau Beta Pi; Member Omicron Delta Kappa; received the Kamm International Appreciation Award, 1978; chosen Outstanding graduate student in Chemical Engineering by the members of the MU Chapter of Omega Chi Epsilon, 1979; received special recognition from the Vice President for Student Services, 1980.